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A 3D finite-element operational model for waters of Latvian jurisdiction

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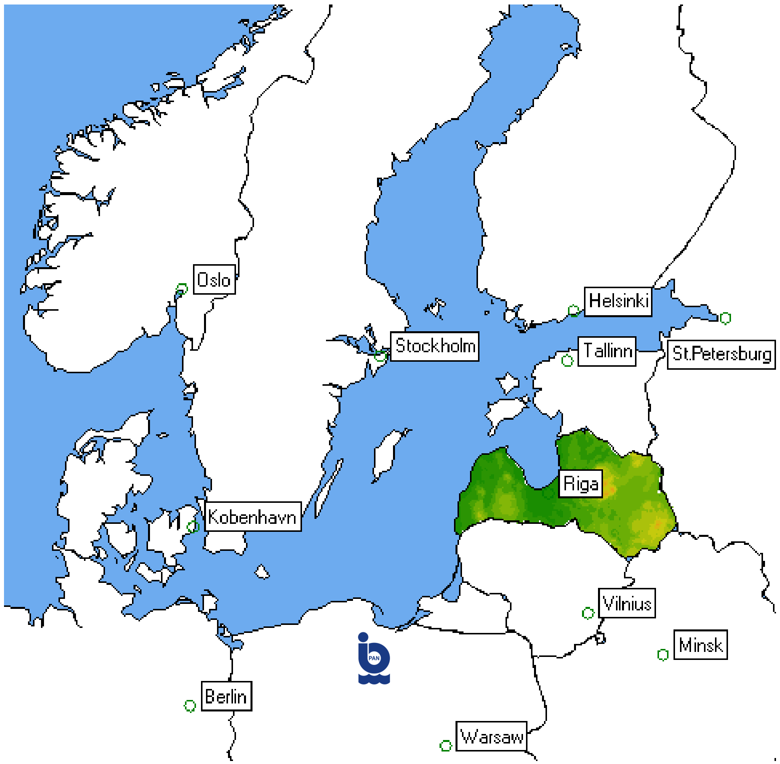
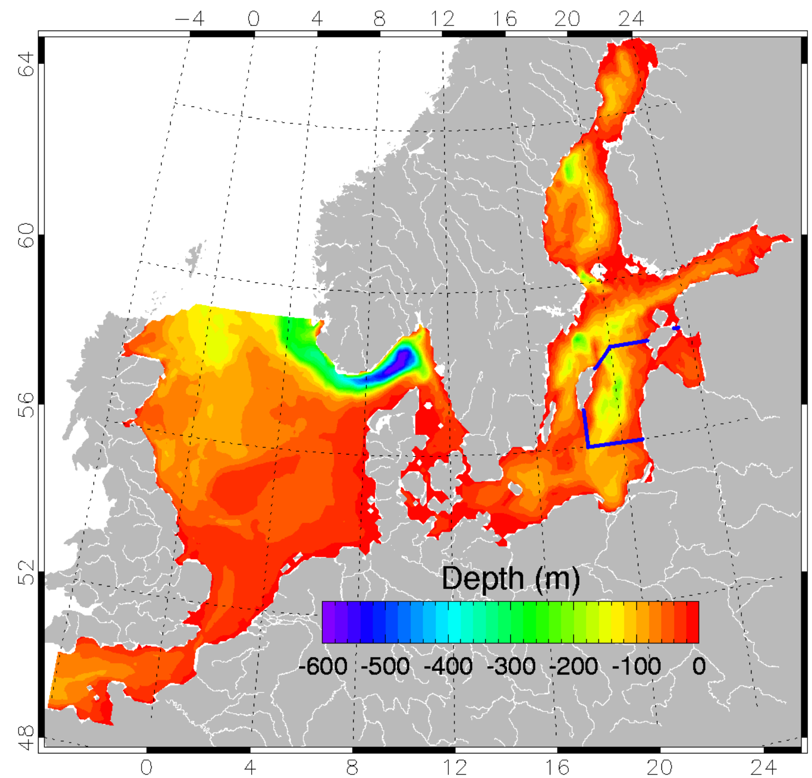
Introduction (1)



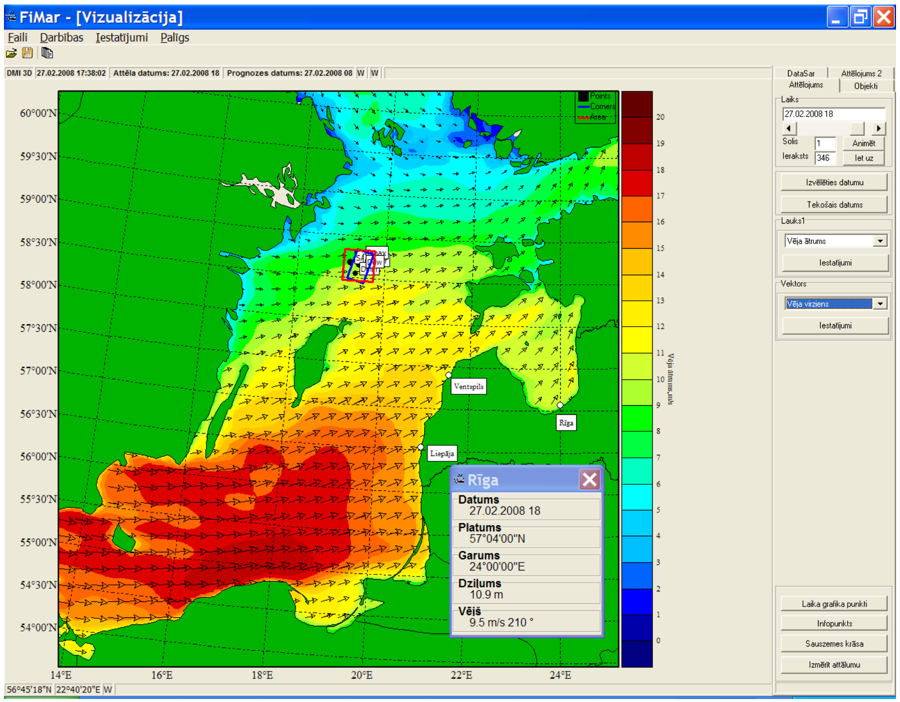


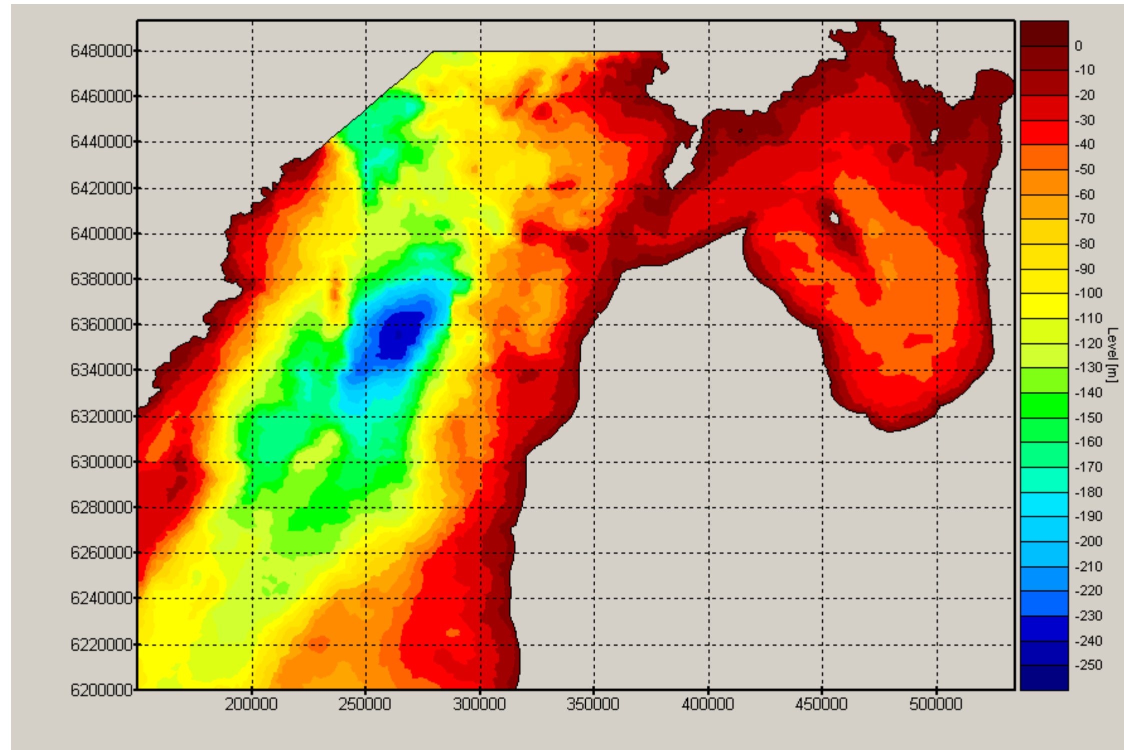
Source: ocean.dmi.dk

Where we are
on the map...

The Danish Meteorological Institute (DMI) routinely runs a suite of models, including a version of BSHcmod (at a resolution of 11 km for the Baltic Sea) and DMI-Hirlam. Sea state forecast is given 60 hours ahead.

Introduction (2)



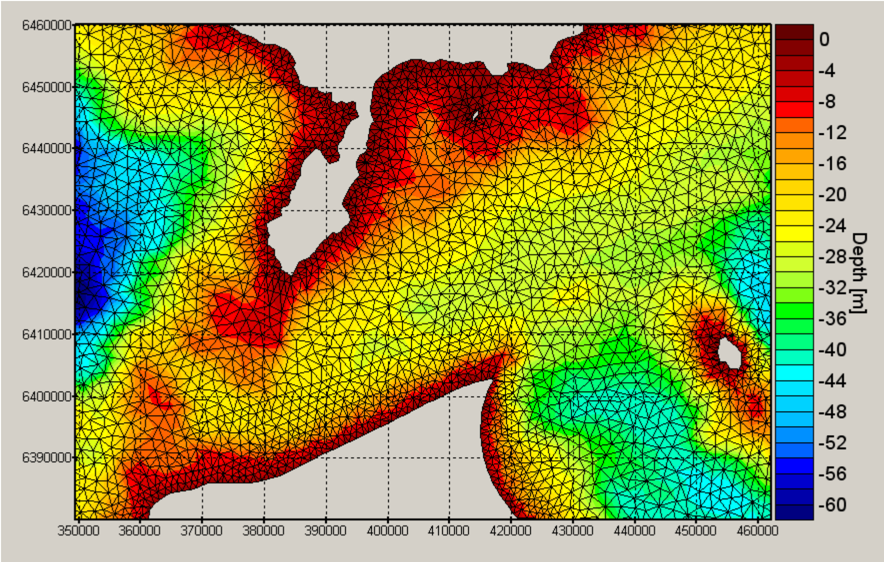


We provide sea-state forecasts and track drifting objects using both Baltic-wide DMI forecasts and output from our nested model FiMar for WLJ.

The bathymetry of the modelling region for the operational model for waters of Latvian jurisdiction (WLJ)

The model for WLJ

- Finite-element (continuous Galerkin) with horizontal resolution of (1–2) km
- Sigma-coordinate
  with 30 equidistant sigma levels
- Free-surface
- Hydrostatic
- Time-split
  no split-out external mode;
  1st order in time;
  with a time step of 450 s



The stabilized level equation (1)

- Both velocity $\vec{U}$ and level s are discretized using the same linear P1-P1 continuous elements (unstaggered mesh, an analogue to the Arakawa A grid).
- $\frac{\partial s}{\partial t} = -\operatorname{div}(h \vec{U}_{\text{avg}})$, where $h = s(x, y, t) - b(x, y)$, and

  $\frac{\partial}{\partial t} \vec{U}_{\text{lev}} = -g \vec{\nabla} s$, where $\vec{U}_{\text{avg}} = \vec{U}_{\text{lev}} +$ constant terms...
- Stabilization is needed because of the spurious modes: Le Roux *et al.*, Mon. Wea. Rev. (1998) 126, 1931.

$$\frac{s^{n+1} - s^n}{\Delta t} = -h^n (\operatorname{div} \vec{U}_{\text{avg}})^{n+1} - \vec{U}^n_{\text{avg}} \cdot \vec{\nabla} s^n$$

$$= -h^n (\operatorname{div} \vec{U}_{\text{avg}})^n \quad - \vec{U}^n_{\text{avg}} \cdot \vec{\nabla} s^n + g h^n (\Delta t) \Delta s^{n+1}$$

$$= -\operatorname{div}(h \vec{U}_{\text{avg}})^n + g h^n (\Delta t) \Delta s^{n+1} \quad \text{(to the 1st order)}$$

The stabilized level equation (2)

- Small-scale level fluctuations (including numerical modes) are suppressed. A quite similar approach was used by Ambrosi et al., J. Hydraul. Egng. 122 (1996), 735.
- The stabilization is consistent at small Δt;
  cf. Hanert *et al.*, Ocean Model. 5 (2002), 17.
- The stabilization becomes relatively unimportant for level fluctuations of a larger scale Δx when
  $$\Delta x \gg \sqrt{gh}\,\Delta t$$
  This is not too restrictive near the shore.
- Recent estimate of the effects of stabilization: Danilov *et al.*, Ocean Dyn. 58 (2008) 365: "*stabilization does not lead to noticeable effects if its strength is kept within certain limits*".

Vertical velocity (1)

- ➔ is diagnosed from the horizontal velocity field by virtue of continuity; one obtains the difference of vertical velocities at adjacent sigma-levels;
- ➔ therefore relates the kinematic boundary conditions (BC) at bottom and surface one to another (the BC's are not independent: having satisfied any of them, the other is recovered automatically);
- ➔ is therefore related to the level evolution.

- The exact correspondence should be observed *exactly* in the discrete formulation; see [White et al., Mon. Wea. Rev. 136 (2008), 420] on the details of the discrete-compatibility issue.

Vertical velocity (2)

So next, integrating Eq.(14) over the vertical and formally substituting s with unity, one obtains:

$$w_n - w_{n-1} = -\frac{\Delta\sigma}{2}\left[\frac{\partial\big(hu_n + hu_{n-1}\big)}{\partial x'} + \frac{\partial\big(hv_n + hv_{n-1}\big)}{\partial y'}\right] + (\sigma_n u_n - \sigma_{n-1}u_{n-1})\,\frac{\partial h}{\partial x'} + (u_n - u_{n-1})\,\frac{\partial b}{\partial x'} + (\sigma_n v_n - \sigma_{n-1}v_{n-1})\,\frac{\partial h}{\partial y'} + (v_n - v_{n-1})\,\frac{\partial b}{\partial y'}\,. \quad (15)$$

As w_0 is known from the kinematic boundary condition at the bottom

$$w = u\,\frac{\partial b}{\partial x} + v\,\frac{\partial b}{\partial y} \qquad \text{at } z = b\,, \quad (16)$$

Eq.(15) can be used to obtain w consequently at every σ layer beginning from the bottom one. At $\Delta\sigma$ = const, all versions of Eq.(15) written for $n = 1 \ldots N$ can be summed up to give

$$w_N - w_0 = -\left[\frac{\partial(h{<}u{>})}{\partial x'} + \frac{\partial(h{<}v{>})}{\partial y'}\right] + u_N\,\frac{\partial h}{\partial x'} + (u_N - u_0)\,\frac{\partial b}{\partial x'} + v_N\,\frac{\partial h}{\partial y'} + (v_N - v_0)\,\frac{\partial b}{\partial y'}\,. \quad (17)$$

Taking Eq.(16) and the elevation equation

$$\frac{\partial\zeta}{\partial t} = -\left[\frac{\partial(h{<}u{>})}{\partial x'} + \frac{\partial(h{<}v{>})}{\partial y'}\right] \quad (18)$$

into account, where $<u>$, $<v>$ are the components of the depth-averaged horizontal velocity, the kinematic boundary condition at the surface (13) is exactly restored:

$$w_N = \frac{\partial\zeta}{\partial t} + u_N\,\frac{\partial\zeta}{\partial x'} + v_N\,\frac{\partial\zeta}{\partial y'}\,. \quad (19)$$

(So the kinematic boundary conditions are not independent.) This advantageous property of Eq.(15) persists in its fully discrete version as long as the horizontal discretization of Eqs.(15) and (18) is the same.

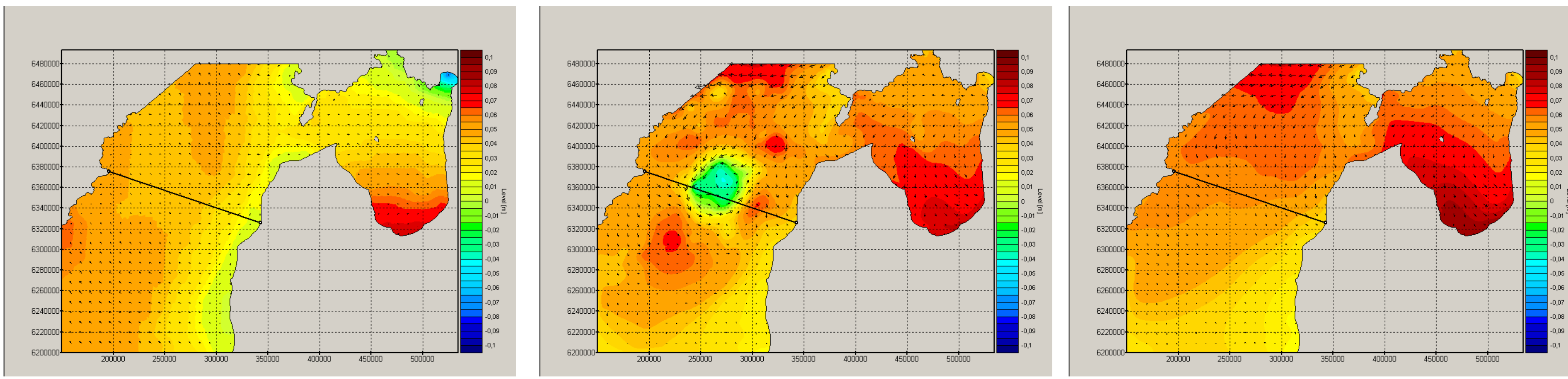
Vertical velocity (3)

6:00 03.06.2008
The initial state

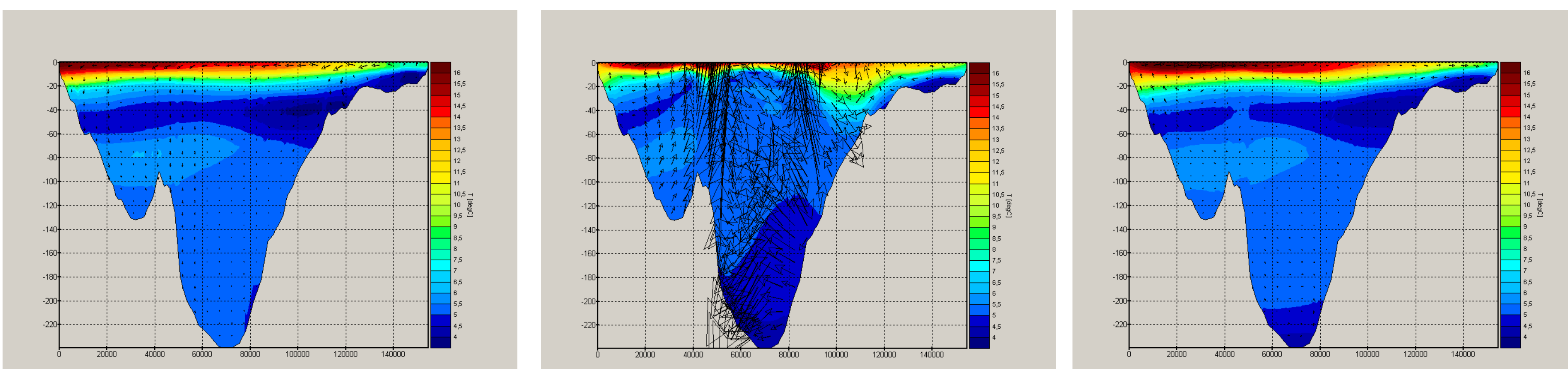
9:00 07.06.2008
The end state

Incompatible **Compatible**

Level



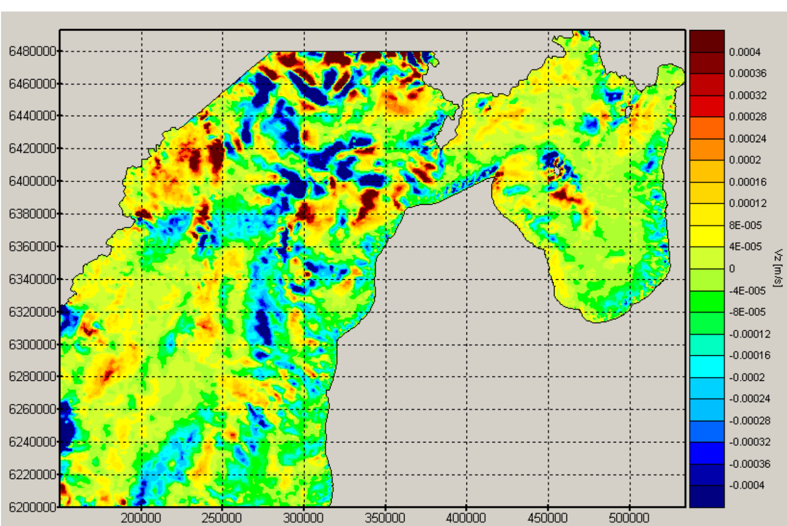
Temperature



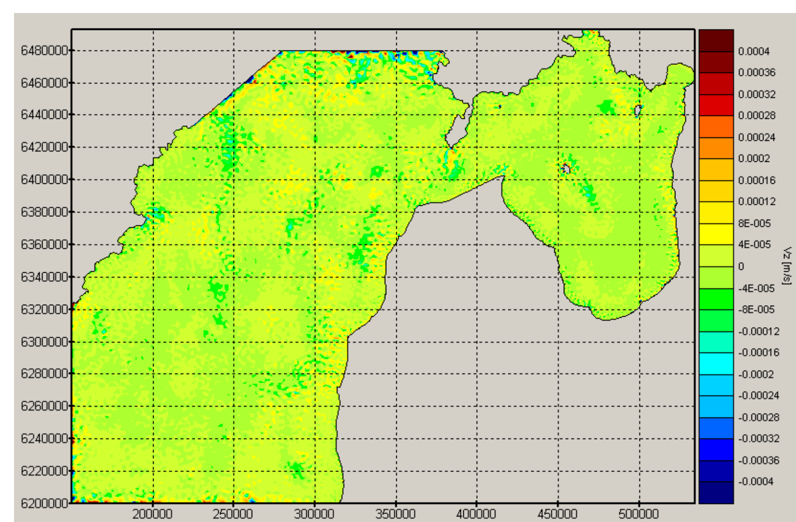
Vertical velocity (4)

- The incompatibility can be (and often is?) masked by adjusting velocities to match *both* BC's.
- The incompatibility can result in artificial currents along the (closed) isobaths in stratified sea similarly to the pressure-gradient problem.

Incompatible code, no buoyancy force



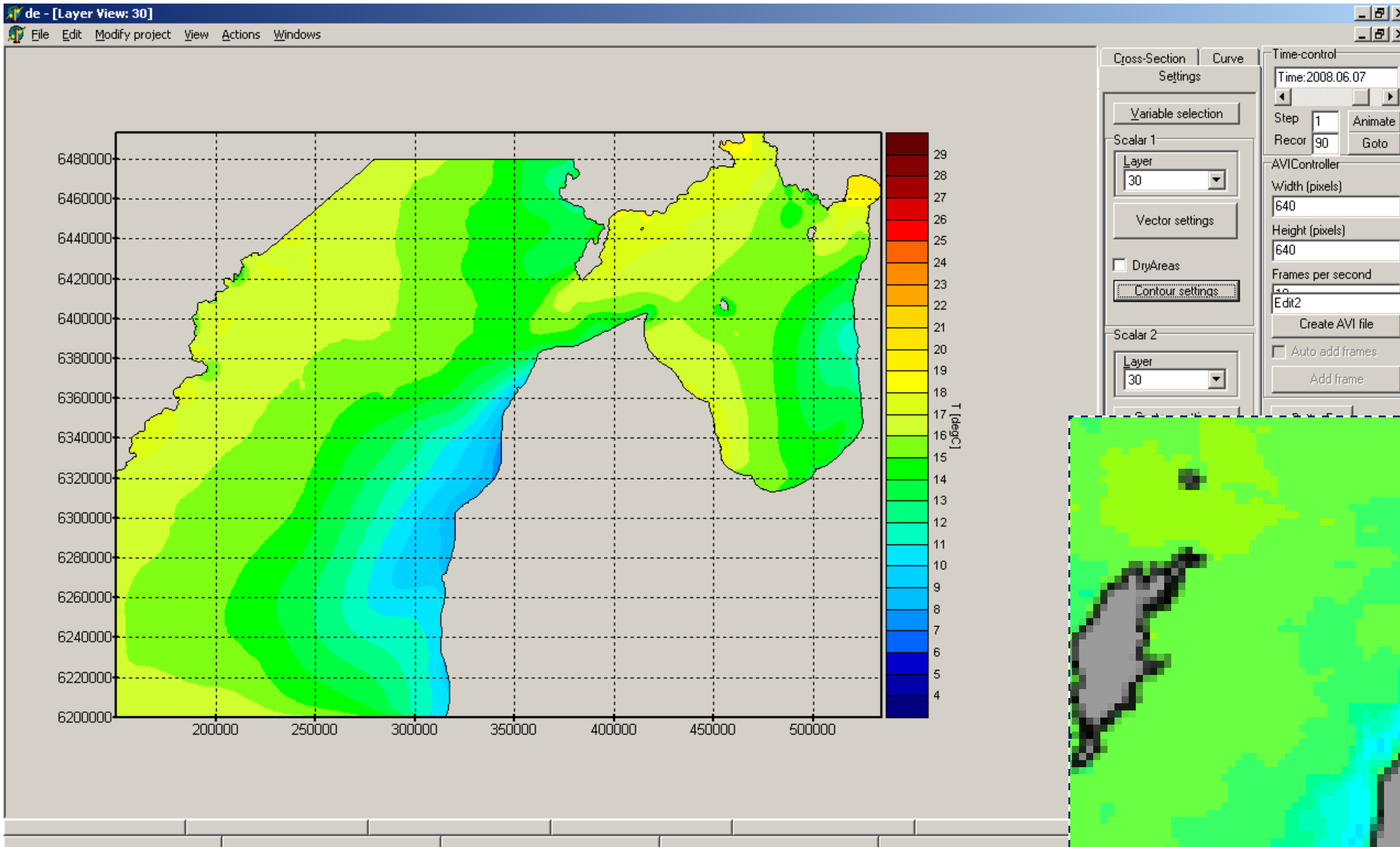
Compatible code with buoyancy force

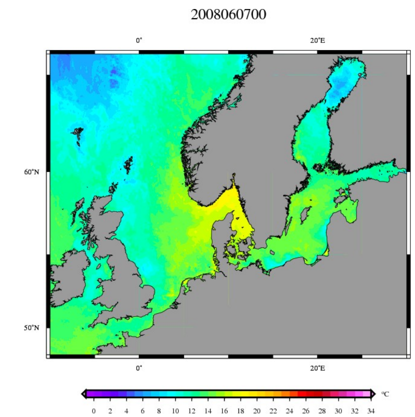


Advection

- A rigorous implementation of the classical streamline-upwind Petrov-Galerkin method (SUPG).
- The evaluation [Budgell et al., Ocean Dyn. 57 (2007) 339] of 24 advection schemes for ocean modelling on unstructured triangular grids revealed that SUPG, due to its robustness, performs quite well in situations where many recent advection schemes just fail.
- Vertical advection needs not to be stabilized.
- Of course, nothing is carried across the σ-surfaces at $\sigma = 0$ or $\sigma = 1$.
- Not yet discretely compatible with the vertical velocity equation [White et al., Mon. Wea. Rev. 136 (2008), 420].

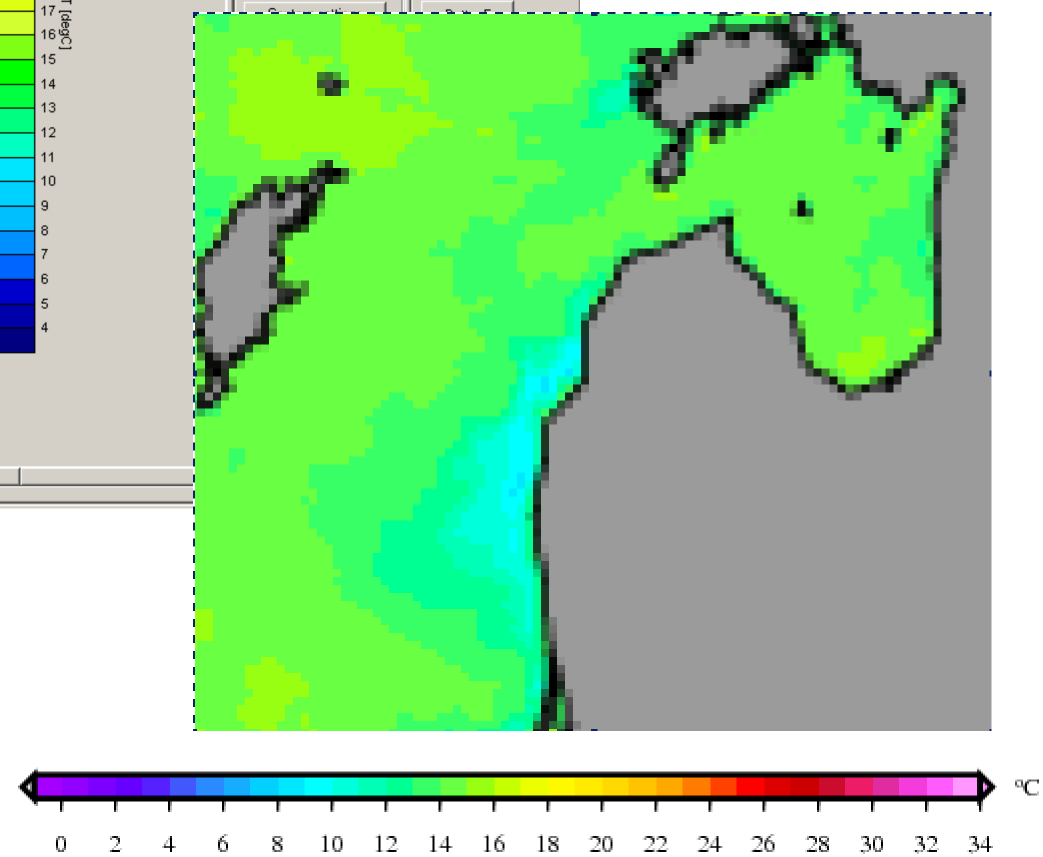
SST: model vs. satellite obs (1)





ocean.dmi.dk

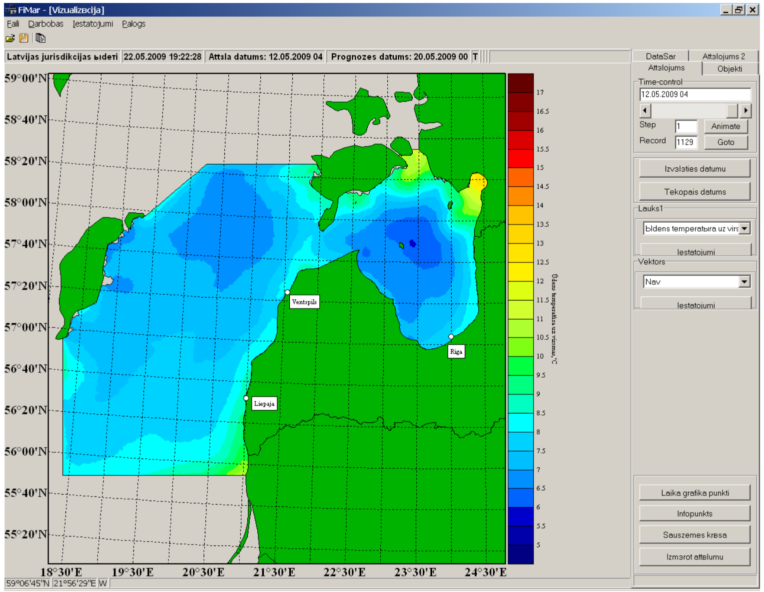
90 hours upon initialization

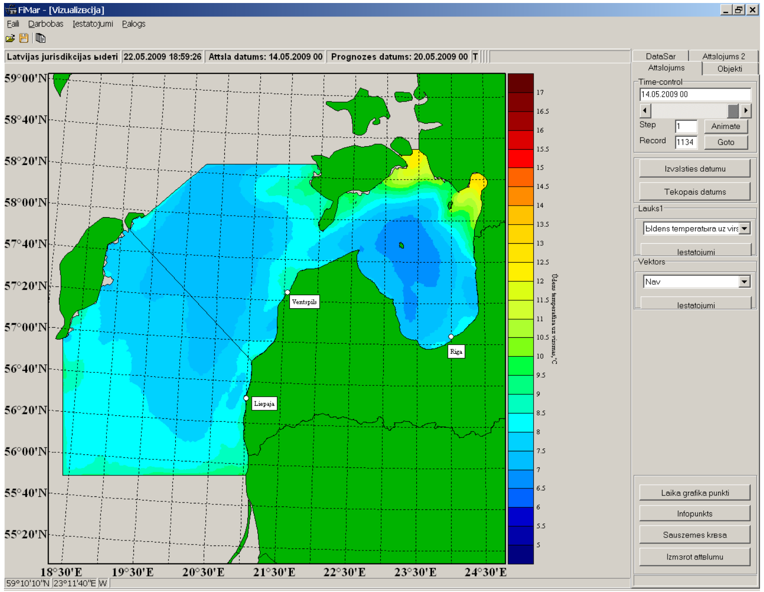


SST: model vs. satellite obs (2): MODIS

May 12, 2009

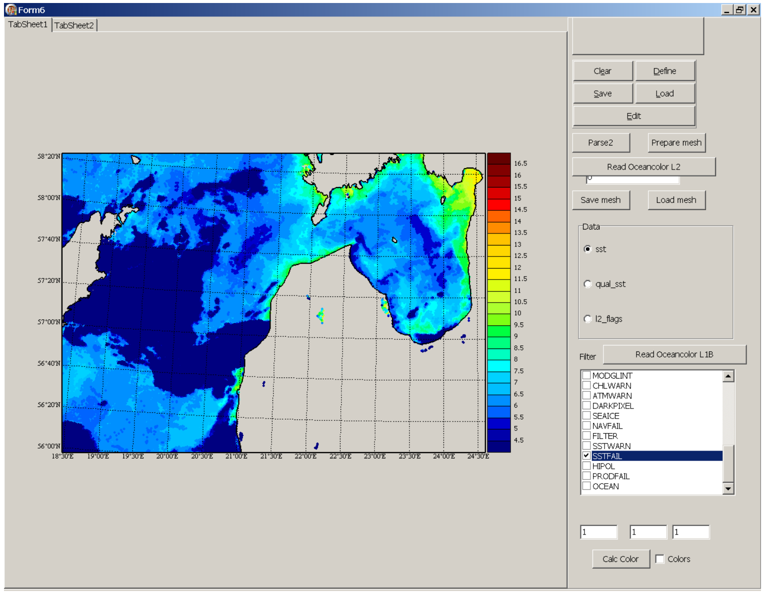
May 13, 2009

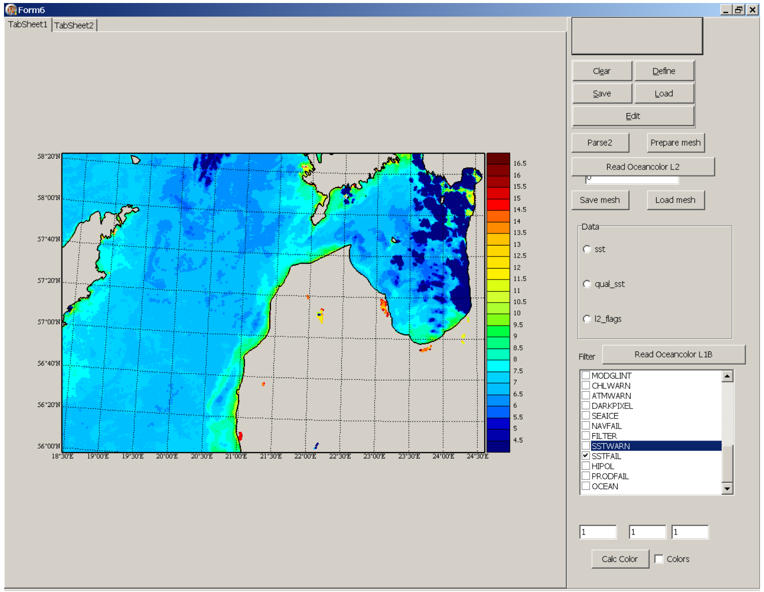




Top row: FiMar, 2-3 weeks upon initialization

Bottom row: oceancolor.gsfc.nasa.gov





The turbulence model (1)

We employ the Mellor-Yamada level 2.5 two-equation q^2-$q^2 l$ model [Mellor & Yamada, 1982] ...

- in its quasi-equilibrium version [Galperin *et al.*, 1988] with the length-scale clipping under stable stratification,
- with enhancements by [Kantha & Clayson, 1994], but with a constant background diffusion of $1 \cdot 10^{-5}\,\mathrm{m}^2/\mathrm{s}$,
- and, optionally, with the enhancements by Craig & Banner and others to take the effect of breaking waves into account (see [Mellor & Blumberg, 2004]).

No convective adjustment.

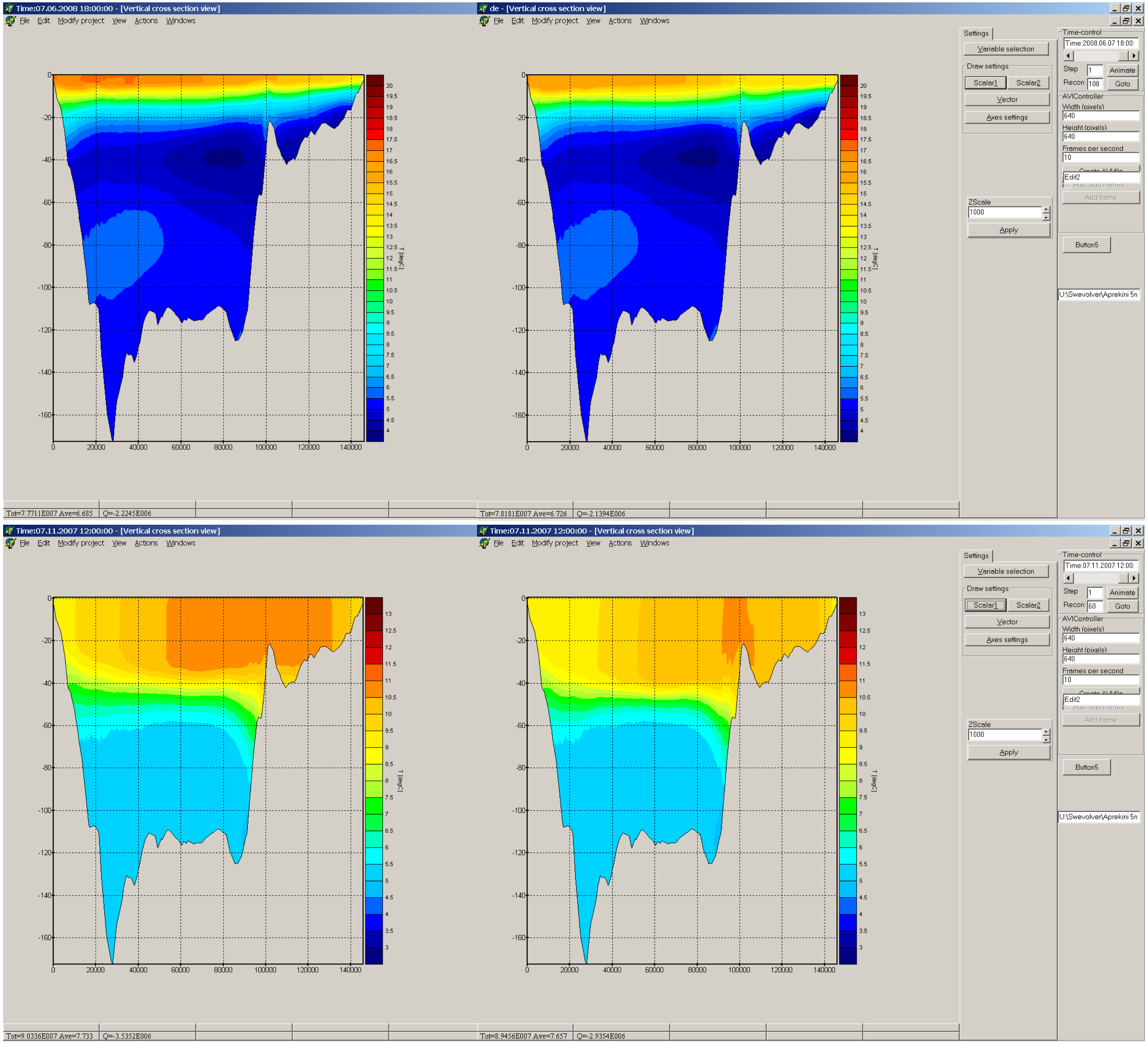
The turbulence model (2)

Original BC

BC due to CB

Jun, 108 hrs

Nov, 60 hrs



About to wrap up...

Must do's: open boundary conditions; horizontal diffusion.

The FiMar operational model for a part of the Baltic Sea has been presented, its build-up and lines of necessary development have been discussed.

Thank you!