Papers

A simple formula for the net long-wave radiation flux in the southern Baltic Sea

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Abstract

This paper discusses problems of estimating the net long-wave radiation flux at the sea surface on the basis of easily measurable meteorological quantities (air and sea surface temperatures, near-surface water vapour pressure, cloudiness). Empirical data and existing formulae are compared. Additionally, an improved formula for the southern Baltic region is introduced, with a systematic error of less than 1 W m^{-2} and a statistical error of less than 20 W m^{-2}.

1. Introduction

The exchange of radiative energy between the atmosphere and the ocean or land surface exerts a great influence on the Earth's climate (Trenberth 1992). The net long-wave radiation flux is an important component of the total radiation budget of the ocean or sea surface.

The net long-wave radiation flux depends on many physical parameters of the atmosphere and the sea, and these complex dependencies are hard to describe precisely in a straightforward form. However, the literature supplies a number of practical algorithms for estimating the net long-wave radiation flux for climate modelling purposes (e.g. Fung et al. 1984 and the papers cited there, Bignami et al. 1995, Woźniak et al. in press; see also Timofeyev 1983 and Trenberth 1992). The principal aim of this paper is to compare the various formulae for the net long-wave radiation flux with empirical data collected in the southern Baltic region. The second aim is to devise a new and more accurate formula based on collected empirical material.

2. Presentation of the physical problem

The net long-wave radiation flux $LW\uparrow\downarrow$ of a sea surface (also denoted in the literature as the net infrared (IR) radiation flux) is the difference between the IR radiation flux from the sea to the atmosphere $LW\uparrow$ and that from the atmosphere to the sea $LW\downarrow$ (see e.g. Dera 1992).

It is generally assumed that the sea (or earth surface) radiates almost in the same way as a black body, i.e. according to the Stefan-Boltzmann law, or more precisely, as a grey body whose total emissivity is given by ε:

$$LW\uparrow = \varepsilon\sigma T_s^4, \tag{1}$$

where ε is usually estimated at between 0.9 and 1, $\sigma = 5.7 \times 10^{-8}$ W m^{-2} K^{-4} is the Stefan-Boltzman constant – see e.g. Garbuny (1965), and T_s is the absolute temperature of the water surface.

The second part of the effective radiation is the long-wave flux emitted by the atmosphere to the sea. To describe this flux using readily measurable quantities, empirical formulae are applied. Generally, for a clear sky, the following equation is applicable:

$$LW\downarrow = \sigma T_a^4(c_1 + c_2 e_a^{c_3}), \tag{2}$$

where e_a is the surface water vapour pressure (in millibars), T_a is the surface absolute temperature of the air, c_1, c_2, and c_3 are empirically determined coefficients.

If the sky is cloudy, the formula becomes more complicated. Clouds increase the long-wave flux reaching the surface, since they are better absorbers of radiation than a clear atmosphere: they are thus better emitters, too. Cloud cover, the type of clouds and their distribution, and the place of the observation all influence this flux. The most common and simplest way of taking clouds into account is to add a correction factor that depends on only one meteorological parameter – the cloudiness C – measured on a scale from 0 to 1.

In the literature, different empirical formulae can be found for the net long-wave radiation flux $LW\uparrow\downarrow$ given as a function of easily measurable meteorological quantities. Such formulae, based on the above scheme, are presented in Table 1. Formulae T1–T8 were obtained for land environments and the last two (T9 and T10) for marine environments. Since the existing formulae differ from one another, both in their analytical form and the selection of empirical coefficients, it is important to assess their applicability to the estimation of the long-wave energy budget of marine basins in different regions.

Table 1. Formulae for the net long-wave radiation flux at sea surface

References	Formula	
Brunt (1932)	$\varepsilon\sigma T_s^4(0.39 - 0.05e_a^{1/2})(1 - 0.8\,C)$	(T1)
Anderson (1952)	$\varepsilon\sigma(T_s^4 - T_a^4(0.74 + 0.0049e_a))(1 - 0.8\,C)$	(T2)
Berliand & Berliand (1952)	$\varepsilon\sigma T_a^4(0.39 - 0.05e_a^{1/2})(1 - 0.8\,C) + 4\varepsilon\sigma T_a^3(T_s - T_a)$	(T3)
Efimova (1961)	$\varepsilon\sigma T_a^4(0.254 - 0.00495e_a)(1 - 0.8\,C)$	(T4)
Swinbank (1963)	$\varepsilon\sigma(T_s^4 - 9.36 \times 10^{-6}T_a^6)(1 - 0.8\,C)^*$	(T5)
Clark et al. (1974)	$\varepsilon\sigma T_s^4(0.39 - 0.05e_a^{1/2})(1 - 0.69\,C^2) + 4\varepsilon\sigma T_s^3(T_s - T_a)$	(T6)
Bunker (1976)	$0.22(\varepsilon\sigma T_a^4(11.7 - 0.23e_a)(1 - 0.8\,C)) + 4\varepsilon T_a^3(T_s - T_a)$	(T7)
Hastenrath & Lamb (1978)	$\varepsilon\sigma T_s^4(0.39 - 0.056q^{1/2})(1 - 0.53\,C^2) + 4\varepsilon\sigma T_s^3(T_s - T_a)^{**}$	(T8)
Bignami et al. (1995)	$\varepsilon\sigma T_s^4 - (\sigma T_a^4(0.653 - 0.00535e_a))(1 + 0.1762\,C^2)$	(T9)
Woźniak et al. (in press)	$\varepsilon\sigma T_s^4(0.39 - 0.0077e_a)(1 - 0.75\,C^2) + 4\varepsilon\sigma T_s^3(T_s - T_a)$	(T10)

*The emissivity of a water surface $\varepsilon = 0.98$ was taken from Bignami et al. (1995). Only in the model by Woźniak et al. (in press) was this coefficient taken to be 0.95.
**In this case the authors used the specific humidity q instead of the water vapour pressure e_a.

3. Empirical material and methods

The measurements were carried out during 8 cruises of r/v 'Oceania' (PAS) in the southern Baltic region between June 1999 and October 2000. The long-wave radiation fluxes $LW\uparrow$ and $LW\downarrow$ were measured directly with two Kipp & Zonen pyrgeometers (CG1) capable of detecting radiation from 5000 to 25 000 nm. They were mounted on a special boom 6 metres ahead of the ship's bows and 5 metres above the sea surface. The sensors were so placed in order to minimise the influence of the ship's presence on the measurement process. The temperatures of the air and sea surface were measured with two digital thermometers, one being mounted at a height of 5 metres, the other being allowed to drift freely at the sea surface. Radiation data and temperatures were collected on a continuous basis. Additionally, the following meteorological parameters were measured intermittently: the water vapour pressure was measured at a height of 5 metres with a psychrometer, and the cloudiness was estimated from a combination of direct observation and digital images of the sky. For the purposes of comparison the fluxes and temperature data were averaged over 10-minute intervals, which corresponded to the times of the weather observations. Over 500 observations were obtained in this way (including over 200 for clear skies).

The long-wave radiation fluxes from the sky ranged from 170 to 380 W m^{-2}, those from the sea surface from 260 to 440 W m^{-2}. The air temperatures varied from –0.5°C to 20°C, the sea surface temperatures from 2°C to 20°C. The surface water vapour pressure lay within the 4–19 mbar range, and the cloudiness ranged from 0 to 1. The mean values of the measured quantities for each cruise are presented in Table 2, and the value distribution frequency is given in Fig. 1.

Table 2. Mean values of the physical quantities measured during each cruise

Cruise period	T_s	T_a	T_s–T_a	e_a	C	$LW\uparrow$	$LW\downarrow$	$LW\uparrow\downarrow$
	[°C]			[mbar]		[W m^{-2}]		
01–04.06.1999	13.7	14.7	–1.0	14.6	0.53	376	305	70
04–11.09.1999	19.2	18.9	0.3	17.6	0.31	406	310	95
04–10.08.1999	16.1	12.5	3.6	11.4	0.70	388	313	75
15–27.02.1900	2.4	1.9	0.5	6.3	0.69	320	253	66
12–15.03.2000	2.3	2.2	0.1	5.8	0.80	319	262	56
08–14.05.2000	7.9	9.9	–2.0	10.5	0.34	346	273	73
21–30.09.2000	14.5	14.1	0.4	13.2	0.34	381	299	82
19–22.10.2000	10.4	10.4	0.0	10.6	0.21	351	271	80

T_s – surface temperature, T_a – air temperature, T_s–T_a – the difference between the two, e_a – water vapour pressure, C – cloudiness, $LW\uparrow$ – upward long-wave radiation flux emitted by the ocean, $LW\downarrow$ – atmospheric long-wave radiation reaching the surface, $LW\uparrow\downarrow$ – net long-wave radiation flux at sea.

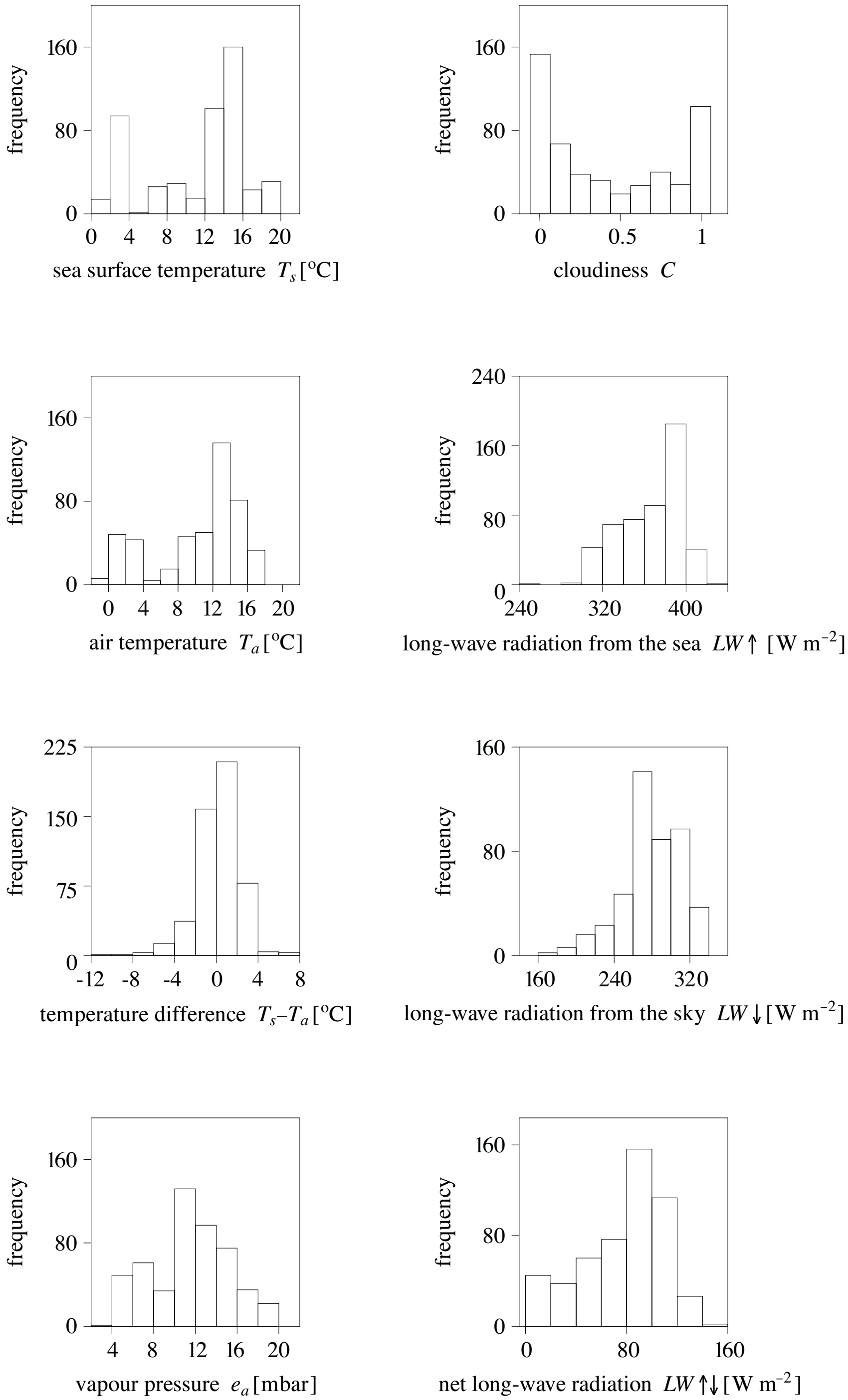

Fig. 1. Distributions of the measured physical quantities

4. Empirical verification of existing formulae

The meteorological data were substituted in the formulae given in Table 1, and predicted net long-wave radiation fluxes $LW\uparrow\downarrow_{\text{model}}$ (model were calculated. The testing of the formulae involved comparing the predicted values with the measured ones (denoted as $LW\uparrow\downarrow_{\text{real}}$) obtained as described above. The discrepancies between the results are presented on the basis of an analysis of statistical and systematic errors and correlation coefficients (see Table 3).

Table 3. The systematic and statistical errors of the formulae, and correlation coefficients

References	Systematic error $\langle\varepsilon\rangle$ [W m^{-2}]	Statistical error σ_ε [W m^{-2}]	Correlation coefficient r
Brunt (1932)	–24.4	22.3	0.73
Anderson (1952)	–27.9	21.9	0.74
Berliand & Berliand (1952)	–23.2	22.0	0.74
Efimova (1961)	–30.4	22.7	0.71
Swinbank (1963)	–19.0	22.8	0.73
Clark et al. (1974)	–14.2	21.0	0.76
Bunker (1976)	–24.9	22.1	0.73
Hastenrath & Lamb (1978)	41.7	23.6	0.75
Bignami et al. (1995)	6.4	21.2	0.76
Woźniak et al. (in press)	2.8	21.8	0.78

where
$\varepsilon = LW\uparrow\downarrow_{\text{model}} - LW\uparrow\downarrow_{\text{real}}$,
$\langle\varepsilon\rangle$ – arithmetic mean of errors (systematic error),
σ_ε – standard deviation of errors (statistical error),
$r = \frac{\langle LW\uparrow\downarrow_{\text{real}}\ LW\uparrow\downarrow_{\text{model}}\rangle - \langle LW\uparrow\downarrow_{\text{real}}\rangle\ \langle LW\uparrow\downarrow_{\text{model}}\rangle}{\sigma_{\text{real}}\,\sigma_{\text{model}}}$ – correlation coefficient.

As can be seen, the systematic errors in the formulae differ significantly over the range from –30.4 to 41.7 W m^{-2}. This means that the net long-wave radiation flux has been considerably over- or underestimated. The formulae of Bignami et al. (1995) and Woźniak et al. (in press) are encumbered with the smallest systematic errors. This is probably because only these formulae were specifically derived for marine environments.

All the formulae discussed here are encumbered with significant statistical errors, which exceed 21 W m^{-2}. The correlation coefficient between real and predicted values can be used as another statistical criterion; this is rather low and varies from 0.71 to 0.78.

If all three statistical criteria are taken into account, the majority of the formulae discussed here are in poor agreement with the empirical data obtained for the southern Baltic region (see also Zapadka & Woźniak 2000, Woźniak et al. 2001). All these formulae were derived for various regions with diverse atmospheric conditions, e.g. prevailing vertical humidity profiles or cloud cover type. Each particular formula probably works well only in the conditions for which it was derived, hence the need for a new formula for the southern Baltic region.

5. A new formula

We attempted to find the best approximation for the net long-wave radiation flux data under the same physical assumptions as had been used previously. The initial assumption was that the sea radiates like a grey body – see eq. (1). The application of correlation techniques showed the total emissivity to be the same as that used in most of the cited formulae and equal to 0.98. The comparison between the modelled and measured long-wave fluxes from the sea $LW\uparrow$ is presented in Fig. 2. In this case, the correlation coefficient between the modelled and measured values is very high: $r = 0.98$.

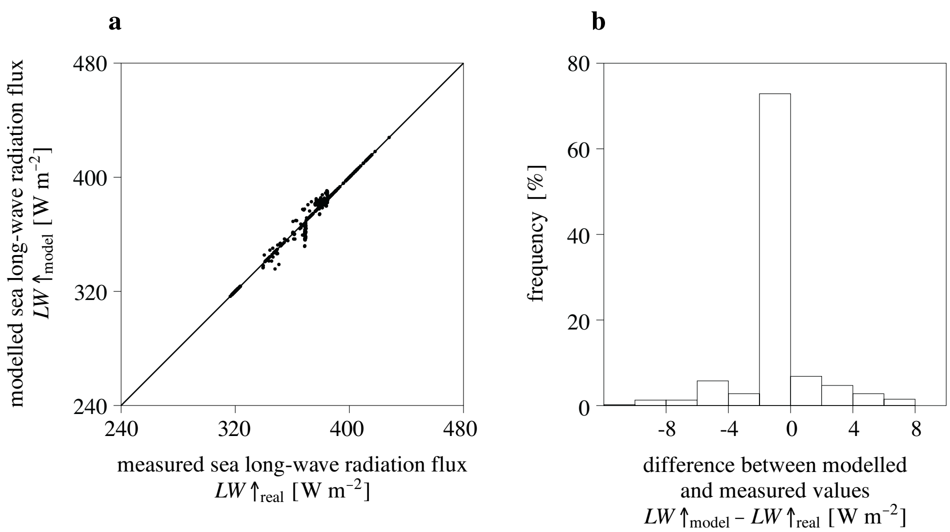

Fig. 2. Comparison between modelled and measured values of long-wave radiation fluxes of the sea surface (a) and error histogram (b)

The second step was to analyse the behaviour of the ratio of the sky radiation $LW\downarrow$ to σT_a^4 for variations in water vapour pressure under clear skies. The data spread in Fig. 3 did not suggest which type of function

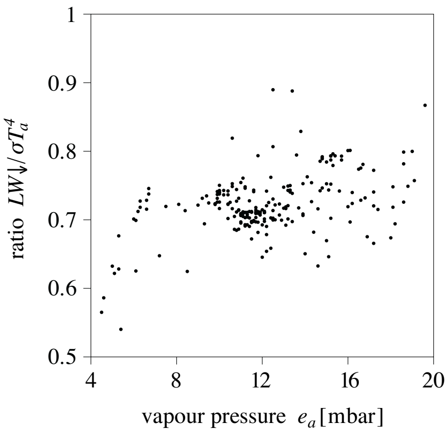

Fig. 3. Values of the ratio $LW\downarrow/\sigma T_a^4$ versus values of vapour pressure e_a for clear sky conditions

could be adopted as an approximate relation between $LW\downarrow$ and e_a. We analysed different types of functions, including those given by eq. (2). The chosen functional form results in

$$LW\uparrow/\sigma T_a^4 = f_1(e_a) = c_1(1 - \exp(c_2 e_a)), \tag{3}$$

where $c_1 = 0.743$ and $c_2 = 0.358$. This function is presented in Fig. 4 together with the mean values of the ratio $LW\downarrow/\sigma T_a^4$ calculated for 14 water vapour pressure intervals. The comparison of the values modelled using function (3) with measured values of $LW\downarrow$ for clear skies is given in Fig. 5. In this case

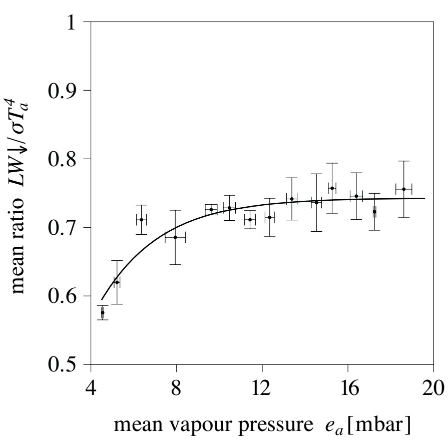

Fig. 4. Mean values of ratio $LW\downarrow/\sigma T_a^4$ versus mean values of vapour pressure e_a for clear sky conditions. The solid line represents the function $f_1(e_a)$ (for details see text)

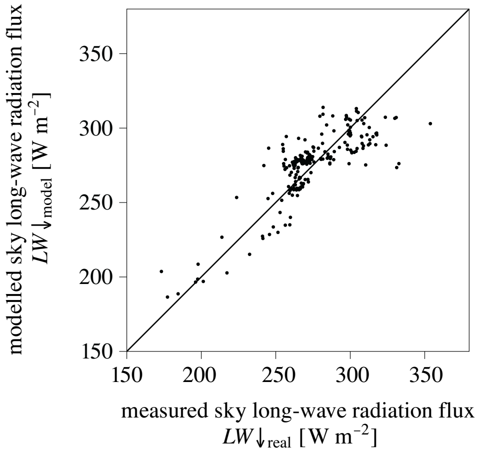

Fig. 5. Comparison between modelled and measured values of the long-wave radiation fluxes of the atmosphere for clear sky conditions

the systematic error is 1 W m^{-2}, the statistical error $\sigma = 15.3$ W m^{-2}, and the correlation coefficient is $r = 0.82$.

The next step was to perform the analogous regression for cloud cover. In this case the linear and quadratic functions were tested. As a result of correlation analysis the dependence of the ratio $LW\downarrow/\sigma T_a^4 f_1(e_a)$ on cloudiness C can be estimated with the quadratic function

$$LW\downarrow/\sigma T_a^4 f_1(e_a) = f_2(C) = 1 + c_3 C + c_4 C^2, \tag{4}$$

where $c_3 = -0.052$ and $c_4 = 0.306$. This function is shown in Fig. 6, and the modelled values of long-wave radiation from the atmosphere $LW\downarrow$ for all

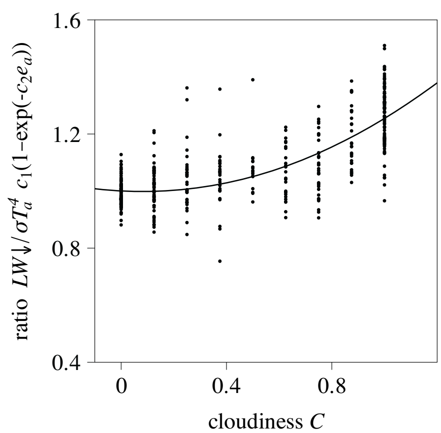

Fig. 6. Values of the ratio $LW\downarrow/\sigma T_a^4 c_1(1 - \exp(-c_2 e_a))$ versus cloudiness C. The solid line represents the function $f_2(C)$ (for details see text)

ranges of cloud cover are compared in Fig. 7. The systematic error is 0.8 W m^{-2}, the statistical error $\sigma = 21.9$ W m^{-2}, and the correlation coefficient $r = 0.83$.

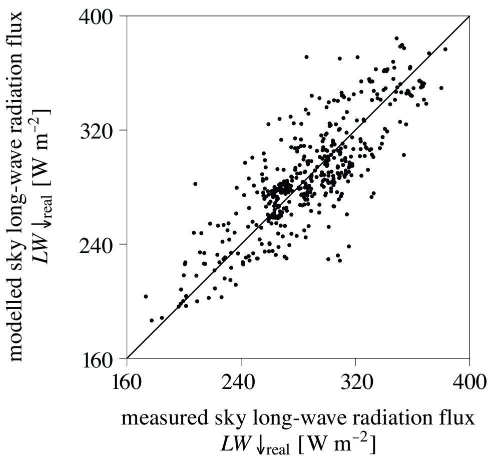

Fig. 7. Comparison between modelled and measured values of the long-wave radiation fluxes of the atmosphere for all sky conditions

As a summary of the results of the partial estimation, the functional form for the net long-wave radiation flux can be written as

$$LW\uparrow\downarrow = \varepsilon\sigma T_s^4 - \sigma T_a^4 f_1(e_a) f_2(C). \tag{5}$$

The correlation analysis was performed a second time with the use of the full functional form (5) together with all the long-wave radiation data. Summarised results of those approximations are presented in Table 4. As can be seen, the second approximation yields better values of the statistical error and the correlation coefficient, and the proposed formula is as follows:

$$LW\uparrow\downarrow = 0.98\sigma T_s^4 - \sigma T_a^4(0.732(1-\exp(-0.47e_a)(1-0.067\,C+0.301\,C^2)). \tag{6}$$

Table 4. Coefficients of the new formulae, and error analysis

Approximation number	Coefficients c_1	c_2	c_3	c_4	Systematic error $< \varepsilon >$ [W m^{-2}]	Statistical error σ_ε [W m^{-2}]	Correlation coefficient r
1	0.743	0.358	–0.052	0.307	0.1	21.1	0.77
2	0.732	0.476	–0.068	0.302	0.9	19.9	0.79

This is a new formula for calculating the net long-wave radiation flux as a function of readily measurable meteorological quantities for conditions in the southern Baltic Sea.

The correlation between the experimental data, the data calculated with the new formula and the histogram of errors are compared in Fig. 8. The systematic error of the proposed approximation is reduced to 0.9 W m^{-2}, the statistical error is 19.9 W m^{-2}, and the correlation coefficient equals 0.79. These values are better than those given by other, previously analysed formulae.

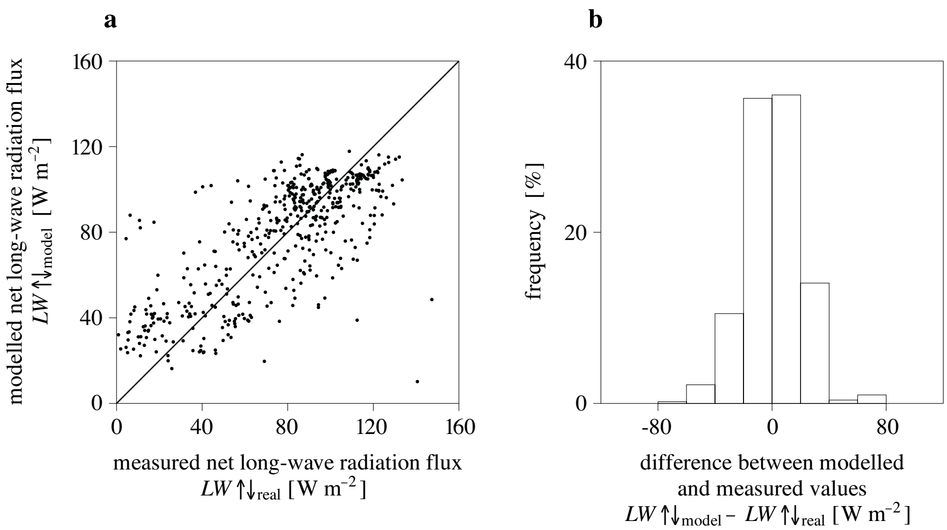

Fig. 8. Comparison between modelled and measured values of the long-wave radiation fluxes of the sea (a); error histogram (b)

6. Some conclusions

The results of this analysis show that the majority of existing formulae obtained for terrestrial environments (eqs. T1–T8) are inappropriate for estimating the net long-wave radiation flux over the southern Baltic because of the very considerable systematic error. The last three formulae obtained for marine environments (eqs. T9, T10 and 6) are more useful for the southern Baltic region, especially the last one, proposed by the authors. However, all the analysed formulae are encumbered with a significant statistical error. In order to obtain more accurate results, the net long-wave radiation flux should in future be estimated with the use of more appropriate environmental parameters describing the type and distribution of clouds, as well as the vertical distribution of water vapour pressure in the atmosphere.

It is also possible to devise new methods of precisely estimating the net long-wave radiation flux of the Baltic Sea based on environmental parameters measured with remote sensing techniques. The authors are at present investigating these very problems.

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