

Characteristic of turbulent horizontal heat exchange in subsurface layers of the Southern Ocean. Part I—Atlantic*

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Abstract

Characteristics of turbulent horizontal heat exchange in the subsurface layer of the Atlantic Ocean have been prepared on the basis of data from the First GARP Global Experiment (FGGE) in the form of maps of the average surface temperatures of the southern hemisphere edited throughout 1979 with a five-day averaging step. The assumptions and the procedure of numerical calculations are presented. The results of computations are shown in terms of isolines of horizontal heat exchange coefficient for a year period (1979) and for four seasons.

1. Objects, aims and scope of the paper

During the period of 28 December 1978—30 October 1979, the first macroscale oceanographic experiment was accomplished, the aim of which was to explain the role of oceans in the generation of the climatical variations and weather phenomena in the Earth's atmosphere. The experiment, organized under the auspices of the World Meteorological Organization (WMO) and Intergovernmental Oceanographic Commission (IOC-UNESCO), has been known as the Global Weather Experiment or, more often, the First GARP Global Experiment (FGGE), within the framework of which a special system of acquiring data on the temperature of oceanic upper layer was developed, employing freely drifting automatic oceanographic buoys, as well as satellite methods. The data recorded were collected under the programme of the Integrated Global Oceanic Stations System (IGOSS) by the Canadian Marine Environmental Data Service (MEDS) in Ottawa, thus three data sets being issued

* Part II—Pacific and Indian Ocean will appear in one of the forecoming OCEANOLOGY.

in real time and a five-day averaging period [3]: the maps of oceanographic buoys drifting paths, the maps of mean climatic temperature characteristics, and of the temperature anomalies in the subsurface oceanic layer. Such data elaborated for all three oceans on the southern hemisphere (the Atlantic, Pacific and Indian), including also a section of the Atlantic area, were rendered accessible free of charge to research institutions of several countries in order to support the attempts to explain the role of oceans in the generation of climatic and weather changes. Within this programme and based on the data from the Canadian Service [3], and the maps of surface currents [1], the Institute of Oceanology of the Polish Academy of Sciences at Sopot has elaborated the characteristics of turbulent horizontal heat exchange in the subsurface layer of the Southern Ocean, thus providing the turbulent diffusion coefficients, the relevant characteristic scales of turbulence and temperature heterogeneity variances. The results given as the maps of isolines illustrate annual and four seasonal characteristics relating to the period of the FGGE.

The aim of this work is the extension of oceanographical description to include additional elements referring to the structure of turbulent horizontal heat exchange in the subsurface oceanic layer. The paper gives the description of the research methods and the results of computations carried out for the Atlantic Ocean and presented in the polar coordinate system analogically to the maps of mean temperatures and anomalies elaborated by the Canadian Service. Similar characteristics for the Pacific and Indian Oceans will be published in the subsequent issues of *Oceanology*.

2. Fundamental equations, assumptions and empirical data employed in the determination of the characteristics of turbulent heat exchange

The data made, available by the Service in Ottawa, represent the empirical realization of a space-time random function of subsurface temperatures, $T(x_i, t)$, where x_i is a generalized coordinate of the observation station and t characterizes the time period of the FGGE from 28 December 1978 to 21 December 1979. The discrete quantization step assumed by the Centre in Ottawa for $T(x_i, t)$ in real time, *ie* the time interval between successive maps of the subsurface temperature isolines (Fig. 1), averaged within a period of 120 hours (5 days) amounted to $\Delta t = 120$ hrs (5 days). The discrete space quantization step of function $T(x_i, t)$ assumed by the authors herein, *ie* the distance between the sites where the turbulent exchange characteristics were determined, in the polar coordinate system amounts to: $\varphi = 5^\circ$ along the meridian, and $\lambda = 4.5^\circ$ along the parallel, where φ is the latitudinal angle, λ —the angle of longitude, and $\Delta r = 1$ m along the Earth's radius. In the Cartesian coordinate system x_i , where the y -axis is directed to the north, the x -axis to the east and the z -axis is parallel to the Earth's radius, the quantization is as follows:

$$2.7 \cdot 10^5 \text{ [m]} \leq \Delta x \leq 4.2 \cdot 10^5 \text{ [m]}; \Delta y = 5.56 \cdot 10^5 \text{ [m]} = \text{const}, \Delta z = 1 \text{ [m]} = \text{const}.$$

Since the empirical data refer exclusively to the subsurface water layers at a standard horizon, the authors have assumed the parameters of the turbulent heat

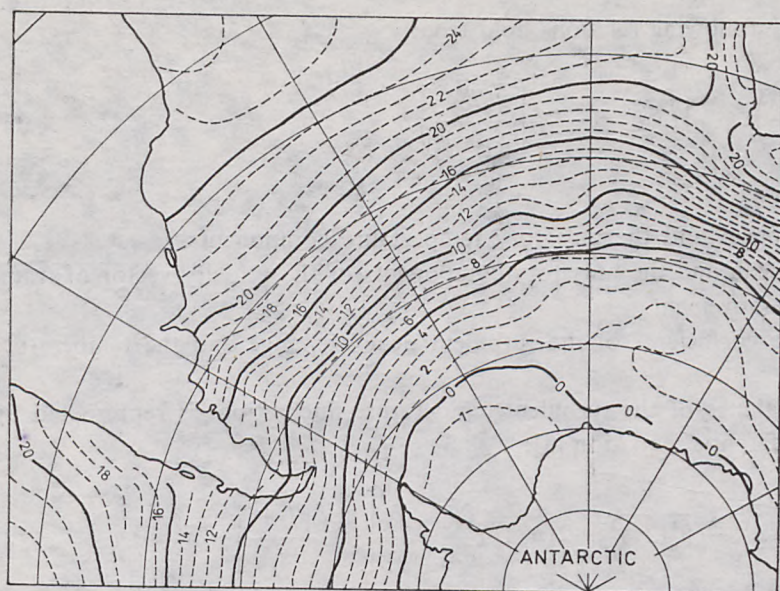


Fig. 1. Exemplary section of a map of temperature isolines (issued by the MEDS Centre in Ottawa [3], data from a five-day averaging period)

exchange determined to be characteristic of the above mentioned layer only, which is illustrated in Figure 2. In this system, the magnitude $\Delta z = 1$ m represents thickness D of the subsurface layer for which the computations were carried out.

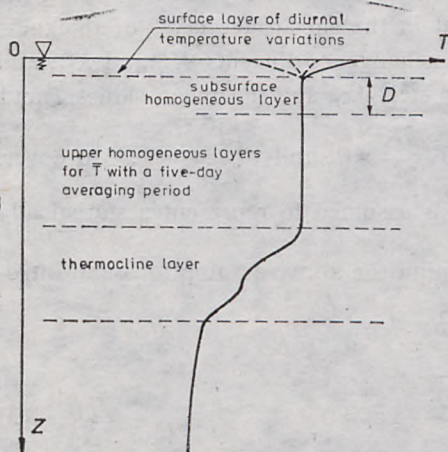


Fig. 2. Schematic temperature distribution with depths of individual layers

The basic assumption in this paper consists in the authors' opinion that in the interval of the space-time discretization and averaging assumed, Boussinesq's hypothesis of flux being proportional to the gradient of the mean value is valid. The authors assumed, as it has been commonly done, that for the surface layer under

consideration the following relations hold true:

$$\overline{u'_i T'} = -K_{ij}^T \left(\frac{\partial \bar{T}}{\partial x_i} \right), \quad \overline{u'_i (T')^2} = -K_{ij}^\sigma \left[\frac{\partial (\overline{(T')^2})}{\partial x_i} \right], \quad (1)$$

where:

\bar{T} and T' —the mean and the pulsatory temperature components,
 \bar{u}_i and u'_i —the mean and the pulsatory components of the velocity vector of water elements movement,

K_{ij}^T and K_{ij}^σ —the coefficients of the turbulent heat exchange and the temperature fluctuation power.

The power balance of the turbulent temperature fluctuations is represented by the following differential equation [8]:

$$\frac{\partial (\overline{(T')^2})}{\partial t} + \bar{u}_i \frac{\partial (\overline{(T')^2})}{\partial x_i} + \frac{\partial}{\partial x_i} [\overline{u'_i (T')^2}] + 2(\overline{u'_i T'}) \frac{\partial \bar{T}}{\partial x_i} - \frac{\partial}{\partial x_i} \left[\mu_T \frac{\partial (\overline{(T')^2})}{\partial x_i} \right] + 2\bar{\varepsilon}_T = 0, \quad (2)$$

where μ_T is the kinematic coefficient of molecular heat diffusion, $\bar{\varepsilon}_T$ is the mean dissipation velocity of turbulent temperature heterogeneities, the bars over the symbols denoting the time averaging.

The authors assumed equation (2) as the basis of the method of calculating the turbulent exchange coefficients.

It can be accepted for the assumed system of numerical time and space quantization and for the data averaging intervals of a quarter and a year, permitting the small-scale interactions to be filtered off, that in the subsurface layer of thickness $D=1$ m (Fig. 2) the values of the turbulent exchange coefficients K_{ij}^T and K_{ij}^σ , and coefficient μ_T are constant within quantization step Δx_i , and the mean values \bar{T} and $\overline{(T')^2}$ change linearly with depth z , ie $\frac{\partial \mu_T}{\partial x_i} = 0$, $\frac{\partial K}{\partial x_i} = 0$, and $\frac{\partial^2 (\overline{(T')^2})}{\partial z^2} = 0$. Moreover, the empirical realization of function $T(x_i, t)$ is assumed to represent a statistically stationary process, ie $\frac{\partial (\overline{(T')^2})}{\partial t} = 0$. Bearing in mind the above assumptions and introducing relation (1) into equation (2) we obtain:

$$\bar{u}_i \frac{\partial (\overline{(T')^2})}{\partial x_i} - K_{ij}^\sigma \frac{\partial^2 (\overline{(T')^2})}{\partial x_i^2} - 2K_{ij}^T \left(\frac{\partial \bar{T}}{\partial x_i} \right)^2 = \mu_T \frac{\partial^2 (\overline{(T')^2})}{\partial x_i^2} + 2\bar{\varepsilon}_T = 0. \quad (3)$$

After writing equation (3) in rectangular (x, y, z) -coordinate system and a suitable rearrangement, a formula enabling the calculation of the turbulent horizontal heat exchange coefficient $K_x = K_y = K_i$ is obtained:

$$K_i^T = \frac{1}{2} \left[\left(\frac{\partial \bar{T}}{\partial x} \right)^2 + \left(\frac{\partial \bar{T}}{\partial y} \right)^2 \right]^{-1} \left\{ \bar{u} \frac{\partial (\overline{(T')^2})}{\partial x} + \bar{v} \frac{\partial (\overline{(T')^2})}{\partial y} + \bar{w} \frac{\partial (\overline{(T')^2})}{\partial z} + \right.$$

$$-K_i^a \left[\frac{\partial^2(\overline{T'})^2}{\partial x^2} + \frac{\partial^2(\overline{T'})^2}{\partial y^2} \right] - 2K_z^T \left(\frac{\partial \bar{T}}{\partial z} \right)^2 - \mu_T \left[\frac{\partial^2(\overline{T'})^2}{\partial x^2} + \frac{\partial^2(\overline{T'})^2}{\partial y^2} \right] + 2\bar{\epsilon}_T \}, \quad (4)$$

where \bar{u} is the parallel, and \bar{v} — the meridional component of the mean velocity of the movement of water elements.

It follows from our investigations that the terms of equation (4), which can be calculated based on the empirical data from the FGGE, assume the following values:

$$\begin{aligned} \left(\left| \frac{\partial \bar{T}}{\partial x} \right|, \left| \frac{\partial \bar{T}}{\partial y} \right| \right) &\leq 10^{-5} \left[\frac{^\circ\text{C}}{\text{m}} \right], \\ 10^{-8} \left[\frac{^\circ\text{C}^2}{\text{m}} \right] &\leq \left(\left| \frac{\partial(\overline{T'})^2}{\partial x} \right|, \left| \frac{\partial(\overline{T'})^2}{\partial y} \right| \right) \leq 10^{-5} \left[\frac{^\circ\text{C}^2}{\text{m}} \right], \\ 10^{-13} \left[\frac{^\circ\text{C}^2}{\text{m}} \right] &\leq \left[\frac{\partial^2(\overline{T'})^2}{\partial x^2} + \frac{\partial^2(\overline{T'})^2}{\partial y^2} \right] \leq 10^{-10} \left[\frac{^\circ\text{C}^2}{\text{m}} \right], \\ 5 \cdot 10^{-9} \left[\frac{^\circ\text{C}^2}{\text{s}} \right] &\leq \left[\bar{u} \frac{\partial(\overline{T'})^2}{\partial x} + \bar{v} \frac{\partial(\overline{T'})^2}{\partial y} \right] \leq 5 \cdot 10^{-6} \left[\frac{^\circ\text{C}^2}{\text{s}} \right]. \end{aligned} \quad (5)$$

Literature data referring to the values that might be assumed in the subsurface layer by vertical velocity components \bar{w} are very scarce. The measurements under real conditions are almost impossible, and the velocity evaluation utilizing the data from the oceanic interior is of little use. There is no doubt that in the surface layer these velocities approach zero and their estimation based on the data from the interior for the upwelling and downwelling zones in the South Atlantic yields values ranging from 10^{-7} to $10^{-5} \left[\frac{\text{m}}{\text{s}} \right]$. The most comprehensive data were published by Bulakov, Denin and Poyarkov [2], according to which in the upper layer of the upwelling zone with a thickness of 10 m in the South Atlantic, vertical velocity assumes the values $10^{-7} \left[\frac{\text{m}}{\text{s}} \right] \leq \bar{w} \leq 10^{-5} \left[\frac{\text{m}}{\text{s}} \right]$, while in the downwelling zone: $10^{-7} \left[\frac{\text{m}}{\text{s}} \right] \leq \bar{w} \leq 10^{-6} \left[\frac{\text{m}}{\text{s}} \right]$. These data, however, are not reliable for the subsurface layer. The latest paper by Jankowski [9] gives the estimation of $\bar{w} \leq 10^{-8} \left[\frac{\text{m}}{\text{s}} \right]$ for homogeneous oceanic surface layer. It seems that this limit is the most reliable for the layer considered. Similarly, the values of the vertical heat exchange coefficient K_z^T , and its relation with the vertical gradient of mean temperature can be estimated in virtue of literature data [5, 6]. When assessing product $K_z^T \left(\frac{\partial \bar{T}}{\partial z} \right)^2$, one should take into consideration the possibility of self-compensation of this relation, *ie* the higher $\frac{\partial \bar{T}}{\partial z}$, the lower coefficient K_z^T , and conversely. According to the majority of papers published recently, for processes in synoptic averaging scales values of $K_z^T \approx 10^{-4}$

$\left[\frac{\text{m}^2}{\text{s}}\right]$ are considered representative for the upper oceanic layer. Such a state of turbulent mixing in the subsurface layer is usually characterized by low values of vertical gradients of mean temperature: $\frac{\partial \bar{T}}{\partial z} \leq 10^{-4} \left[\frac{^\circ\text{C}}{\text{m}}\right]$, and also by low values of variance gradients: $\frac{\partial \overline{(T')^2}}{\partial z} \leq 10^{-2} \left[\frac{^\circ\text{C}^2}{\text{m}}\right]$. Taking into account the above together with relations (5) we can assess the orders of magnitude of the following terms of equation (4):

$$\bar{w} \frac{\partial \overline{(T')^2}}{\partial z} \leq 10^{-10} \left[\frac{^\circ\text{C}^2}{\text{s}}\right],$$

$$K_z^T \left[\frac{\partial \bar{T}}{\partial z}\right]^2 \leq 10^{-12} \left[\frac{^\circ\text{C}^2}{\text{s}}\right],$$

$$\mu_T \left[\frac{\partial^2 \overline{(T')^2}}{\partial x^2} + \frac{\partial^2 \overline{(T')^2}}{\partial y^2}\right] \leq 10^{-17} \left[\frac{^\circ\text{C}^2}{\text{s}}\right],$$

$$\text{where } \mu_T = 10^{-7} \left[\frac{\text{m}^2}{\text{s}}\right].$$

Comparison of these values with those assumed by the sum of advective terms—the last expression in (5)—implies that they are lower by at least one order of magnitude than the sum of advective terms and therefore will be disregarded in our considerations. Equation (4) will be simplified to the following form:

$$K_i^T = \frac{1}{2} \left[\left(\frac{\partial \bar{T}}{\partial x} \right)^2 + \left(\frac{\partial \bar{T}}{\partial y} \right)^2 \right]^{-1} \left\{ \bar{u} \frac{\partial \overline{(T')^2}}{\partial x} + \bar{v} \frac{\partial \overline{(T')^2}}{\partial y} - K_i^\sigma \left[\frac{\partial^2 \overline{(T')^2}}{\partial x^2} + \frac{\partial^2 \overline{(T')^2}}{\partial y^2} \right] + 2\bar{\epsilon}_T \right\}. \quad (6)$$

Obviously, coefficient K_i^σ may differ from K_i^T , but its values, in the assumed intervals of averaging and space-time quantization, can only lie in the range close to that predicted for K_i^T , ie $10^1 \left[\frac{\text{m}^2}{\text{s}}\right] \leq K_i^\sigma \leq 10^4 \left[\frac{\text{m}^2}{\text{s}}\right]$. Moreover, there exists a compensative relation between K_i^σ and $\text{grad} [\overline{(T')^2}]$, similar to that between K_i^T and $\text{grad} [\bar{T}]$. Thus, taking into account the range of values of (5) given for $\text{grad} [\overline{(T')^2}]$ we can see that:

$$K_i^\sigma \left[\frac{\partial^2 \overline{(T')^2}}{\partial x^2} + \frac{\partial^2 \overline{(T')^2}}{\partial y^2} \right] \leq 10^{-9} \left[\frac{^\circ\text{C}^2}{\text{s}}\right]. \quad (7)$$

According to the results obtained so far [7, 9, 10], the mean of dissipation velocity for temperature heterogeneities $\bar{\epsilon}_T$, can be assumed constant within certain intervals of variation of the wave numbers of turbulent structures. Within the anti-

anticipated scales of 10^3 [m] $\leq l \leq 10^6$ [m], the dissipation rate can assume the values in the range of $10^{-9} \left[\frac{^\circ\text{C}^2}{\text{s}} \right] \leq \bar{\varepsilon}_T \leq 10^{-8} \left[\frac{^\circ\text{C}^2}{\text{s}} \right]$. Assuming: $\left(\frac{\partial T'}{\partial z} \right)^2 \approx 10^{-4} \left[\frac{^\circ\text{C}^2}{\text{m}} \right]$ to be a maximum value of mean squares of the temperature pulsation gradients, the horizontal gradients obtained based on empirical data $\left[\left(\frac{\partial T'}{\partial x} \right)^2 + \left(\frac{\partial T'}{\partial y} \right)^2 \right] \leq 10^{-11} \left[\frac{^\circ\text{C}^2}{\text{m}} \right]$ and $\mu_T \approx 10^{-7} \left[\frac{\text{m}^2}{\text{s}} \right]$, the maximum dissipation velocity can be estimated from the following relation:

$$\bar{\varepsilon}_T = \mu_T \left[\left(\frac{\partial T'}{\partial x} \right)^2 + \left(\frac{\partial T'}{\partial y} \right)^2 + \left(\frac{\partial T'}{\partial z} \right)^2 \right] \leq 10^{-11} \left[\frac{^\circ\text{C}^2}{\text{s}} \right]. \quad (8)$$

Comparing the values of (7) and (8) with those assumed by the advective terms in equation (6) we can—with a certain error—adopt a simplified formula for determining the coefficient $K_l^T = K_l$, which takes into account the effect of macro- and mesoscale horizontal circulation of water masses upon horizontal heat exchange processes in the subsurface oceanic layer:

$$K_l = \frac{\bar{u} \frac{\partial (\overline{T'})^2}{\partial x} + \bar{v} \frac{\partial (\overline{T'})^2}{\partial y}}{2 \left[\left(\frac{\partial \bar{T}}{\partial x} \right)^2 + \left(\frac{\partial \bar{T}}{\partial y} \right)^2 \right]}. \quad (9)$$

The dependence between the coefficient K_l and the scale of the process, formulated in its general form by Richardson and analyzed by Ozmidov and Okubo in 1970, has the following form [10]:

$$K_l = \eta \varepsilon_u^{\frac{1}{3}} \cdot l^{\frac{2}{3}}, \quad (10)$$

where:

l —the external scale of the turbulent process (the eddy structure scale) involving cascade energy dissipation and cascade disintegration in the mixing processes,

ε_u —the dissipation velocity for turbulent kinetic energy,

η —a constant coefficient.

It follows from the plot in Figure 3 that the dissipation rate of ε_u assumes, in the range of anticipated values $l \geq 10^3$ m, a constant value enabling simplification of expression (9) to the form $K_l = \beta \cdot l^{\frac{2}{3}}$ (the classical Richardson formula), where $\beta = \eta \varepsilon_u^{\frac{1}{3}} \approx \approx 6.81 \cdot 10^{-4} \left[\text{m}^{\frac{2}{3}} \cdot \text{s}^{-1} \right]$. Comparing the respective sides of equations (9) and (10) we obtain:

$$l = \left\{ \frac{\bar{u} \frac{\partial (\overline{T'})^2}{\partial x} + \bar{v} \frac{\partial (\overline{T'})^2}{\partial y}}{2\beta \left[\left(\frac{\partial \bar{T}}{\partial x} \right)^2 + \left(\frac{\partial \bar{T}}{\partial y} \right)^2 \right]} \right\}^{\frac{3}{2}}. \quad (11)$$

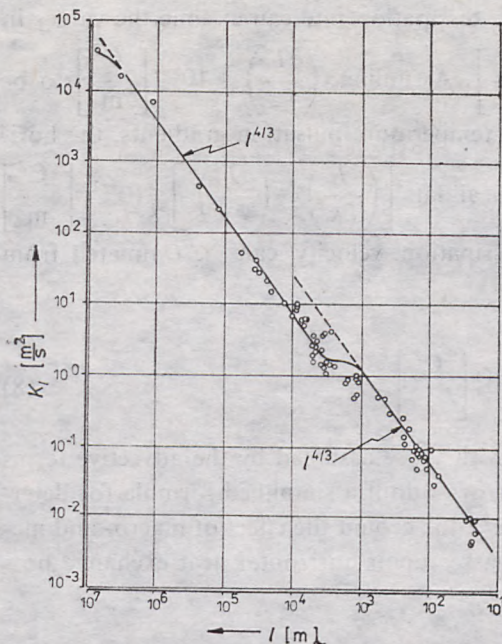


Fig. 3. The dependence of coefficient K_l on the scale of the process (according to Ozmidov and Okubo [12])

In this paper, equations (9) and (11) are fundamental for the assessment of the influence of advective factors upon macro- and mesoscale processes of turbulent horizontal heat exchange in the subsurface oceanic layer.

3. Results of calculations

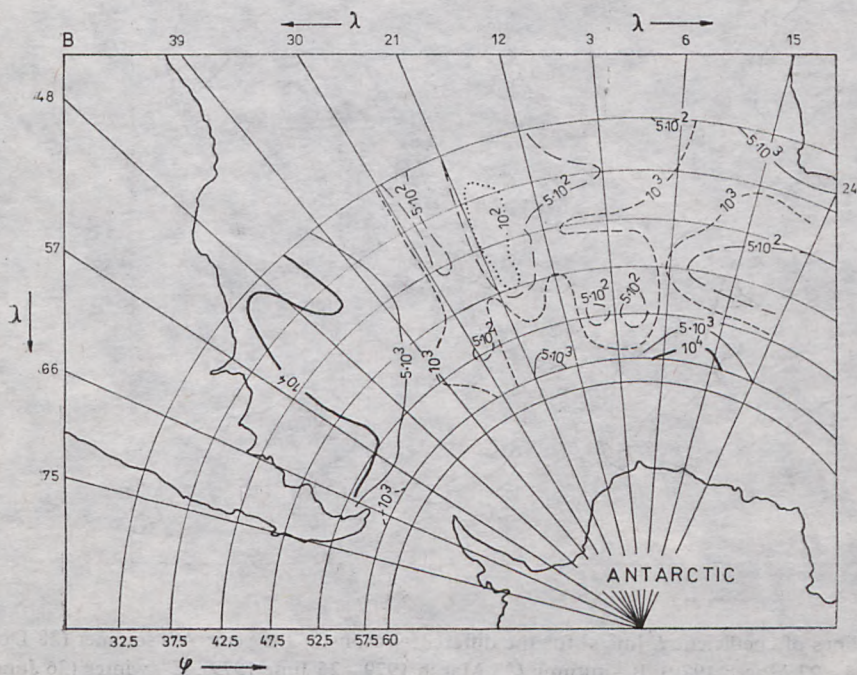
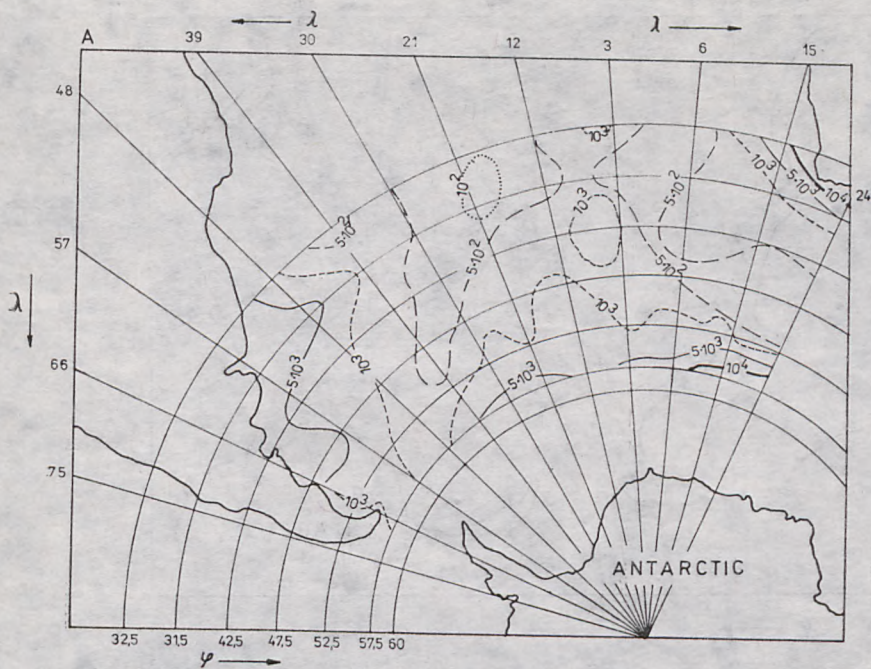
The values of coefficients K_l were computed for four seasonal periods (Figs. 4A-D) on the southern hemisphere:

- (i) 28 December 1978–22 March 1979 (summer),
- (ii) 23 March 1979–25 June 1979 (autumn),
- (iii) 26 June 1979–23 September 1979 (winter),
- (iv) 24 September 1979–27 December 1979 (spring),

and for the annual period (Fig. 5):

- 28 December 1978–27 December 1979 (a year).

The considered area of the South Atlantic lies between the parallels of southern geographical latitude: $30^\circ \leq \varphi \leq 60^\circ$, thus stretching from the Subantarctic to the Antarctic convergence, at the borders undergoing the influence of: Circumantarctic circulation (West Wind Drift) from the south, Falkland Current from the west, and Benguela and Agulhas Currents from the north-east. The effect of these macro-scale circulations on the turbulent heat exchange is clearly seen on the five maps of



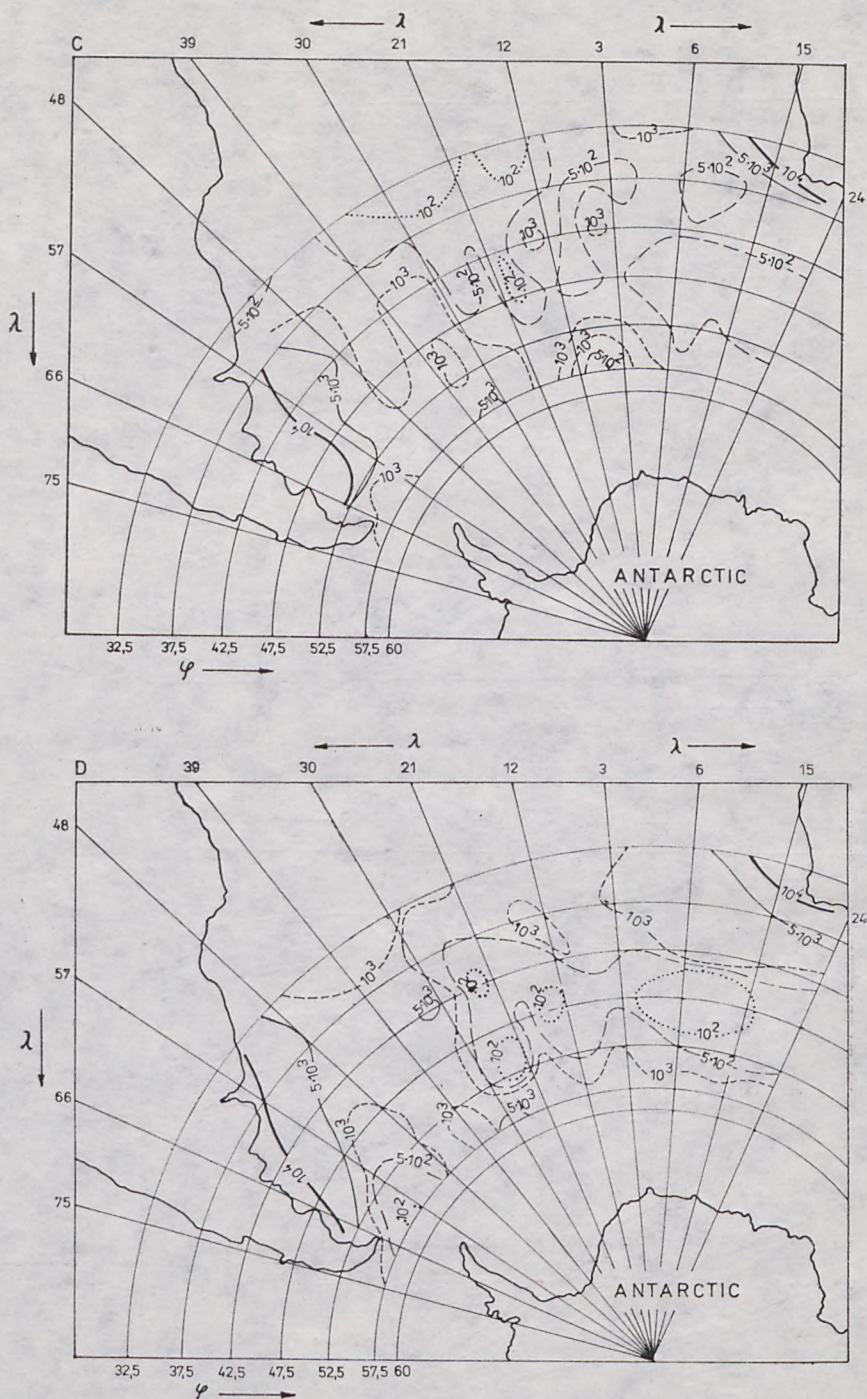


Fig. 4. Isolines of coefficient K_1 [m^2/s] for the different seasons of the year: A – summer (28 December 1978–22 March 1979), B – autumn (23 March 1979–25 June 1979), C – winter (26 June 1979–23 September 1979), D – spring (24 September 1979–21 December 1979)

(\bar{u} and \bar{v}), employed in the computations, have been taken from oceanographic atlas [1] differing to a certain extent from the actual data of 1979. Nevertheless, the authors are of the opinion that even with such a rough approximation the computation results presented are worth consideration when estimating the intensity of macro- and mesoscale horizontal heat exchange in the subsurface oceanic layer involved in the mutual sea-atmosphere interactions taking place during the FGGE.

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