Papers

The effect of the atmospheric pressure field on seasonal Baltic Sea level oscillations

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Abstract

The paper comprises a numerical analysis of a time series of Baltic monthly mean sea levels to determine the seasonal changes. The influence of the atmospheric pressure field on the data analysed was characterized by computing the isolated static effect of the atmospheric pressure and wind-driven level variations. The inverted barometer rule under Baltic sea level conditions was computed and compared with a theoretical formula. The computations were carried out by the use of empirical orthogonal functions (EOF) and a linear dynamic system of stable parameters, as well as estimations by the spectral density and Kalman methods. Basic sea level data were collected from the measurement period 1901–1937.

1. Introduction

In the literature dealing with sea level variations, the seasonal oscillations have commonly been analysed on the basis of monthly mean measurement data. The averaging procedure applied to sea levels measured with a sampling step from 1 to 24 hours is equivalent to low-pass filtration. Although the characteristics of the filter introduced are not absolutely perfect, they are adequate for analysing the phenomenon with the desired accuracy.

The Baltic monthly mean sea levels (BMMSL) were published by Pattulo *et al.* (1955) as the mean values obtained from a large number of measuring stations for long-term observation periods. The BMMSL, summarized in a time series for the period 1901–1940 and calculated on the basis of precisely unified measurement data, were also published (Lazarenko, 1961). The periodic structure of the BMMSL and further computations were based on time series given by this paper.

Fig. 1. Geographical positions of field stations measuring the atmospheric pressure

The effect of the atmospheric pressure field on the oscillations under consideration is of particular importance for Baltic Sea conditions. This effect may be regarded as a static atmospheric influence and a wind-stress forced phenomenon. In the present paper statistical methods have been employed to demonstrate separately the effect of these two factors. The locations of the measuring stations, where the data used in the computations were recorded, are shown in Figure 1.

2. Characteristics and significance of BMMSL in investigations of the Baltic Sea

The monthly mean sea levels were computed from data recorded at 26 measuring stations and levelled to homogeneity (Lazarenko, 1961). It is the only published source giving BMMSL for a forty year period 1901–1940. The trend was eliminated from the data. The data measurement period used was 1901–1937. The BMMSL were determined by both spatial averaging, taking into account onshore measuring stations, and time averaging, by the use of a monthly mean value filter. For the sea levels thus obtained, local oscillations and pulsations with periods shorter than the seasonal variability were practically eliminated. The Baltic mean sea level computed according to these data is −8.2 cm (the zero reference level being in Kronstad), and

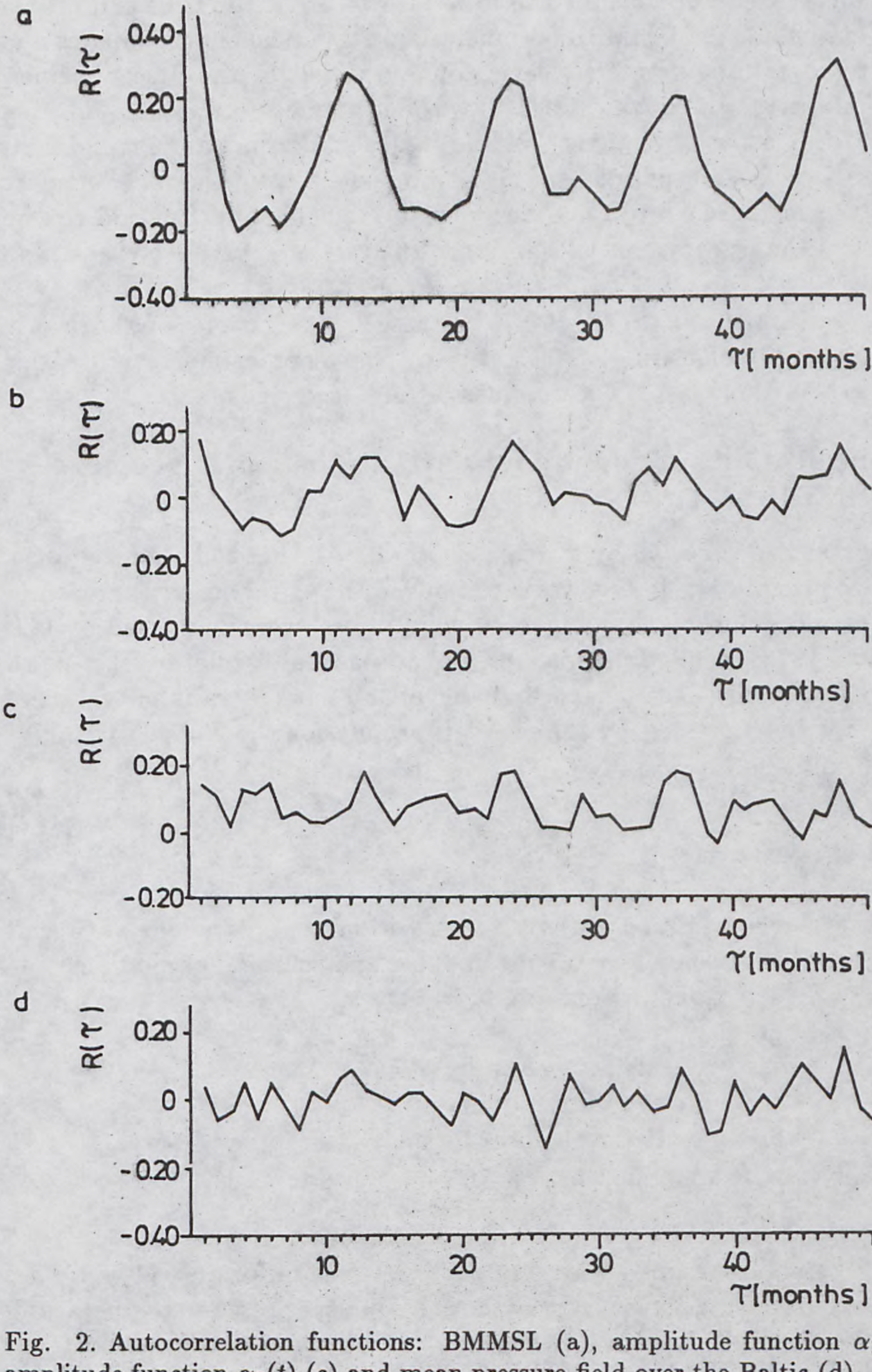

Fig. 2. Autocorrelation functions: BMMSL (a), amplitude function $\alpha_1(t)$ (b), amplitude function $\alpha_2(t)$ (c) and mean pressure field over the Baltic (d)

the oscillations range from −46.8 to 43.1 cm. The standard deviation of the series is 14.76 cm. It should be pointed out that under Baltic Sea, conditions, characteristic of which is the drop in level from the Gulf of Bothnia to the Great Belt, the BMMSL become useful statistical characteristics and do not represent the actual sea level configuration. In the present paper, the periodic structure of the BMMSL was determined from a computation of the autocorrelation functions given in Figure 2. Distinct annual and faint semiannual oscillation periods can be distinguished from the plot. A more demonstrative approach to the occurrence of the semiannual period can be attained by the employment of harmonic analysis or spectral power densities.

The significance of the BMMSL in these investigations of the Baltic consists primarily of getting an insight into the phenomenon of forcing; this applies to the whole basin (local influences are neglected).

3. Computations of the influence of atmospheric pressure on BMMSL

The conclusions drawn by Pattulo *et al.* (1955), that the sea level oscillations are primarily generated by variations in the water density and atmospheric pressure and by the tangential wind stress, were confirmed by Gill and Niiler (1973), who considered the global seasonal oscillations based on the theory of salinity, temperature, sea level and current variability. These findings can be expressed by the general formula for the global variability of the phenomenon

$$\xi_s' = -\frac{p_a'}{g\rho_o} - \frac{1}{\rho_o}\int_{-\mathrm{H}}^{0} \rho' dz + p_b'/g\rho_o, \tag{1}$$

where

$\xi_s = \bar{\xi}_s + \xi_s'$ – the partition of the sea level oscillation into a constant term, represented by the mean value and seasonal oscillations,

p_a – atmospheric pressure, $p_a = \bar{p}_a + p_a'$,

g – gravity,

ρ – sea water density given by $\rho = \bar{\rho} + \rho'$,

ρ_o – constant sea water density,

p_b – pressure at the sea bottom for $z = -H$.

Formula (1) can be simplified to

$$\xi_s' = \xi_{sa}' + \xi_{ss}' + \xi_{sb}', \tag{2}$$

where the terms correspond to the respective terms of formula (1) and describe the components determined by the atmospheric pressure, temperature- and salinity-dependent sea water density, and pressure at the sea bottom. Atmospheric pressure is generally assumed to depress the sea level by 1 cm for a pressure increase of 1 hPa. The effects due to changes in sea-water density are more complex. The relevant tables and calculations can be found in the references. The pressure at the sea bottom is affected by winds generating barotropic flows at high latitudes and causing steric level oscillations by forced vertical thermocline displacements linked with Ekman's circulation. The global seasonal changes are determined principally by ξ'_{sa} and ξ'_{ss}. If the dimensions of sea basins with complex topography at high latitudes are less than those of the oceans, isostatic conditions do not occur, and the influence of the term ξ'_{sa} diminishes. Full isostatic conditions are generally thought to occur on a few-day time scale and in basins whose dimensions are of the order of several thousand miles.

Clearly, under the well-known oceanographic conditions specific to the Baltic, the static influence of the atmospheric pressure does not substantially affect sea level variations. It should, moreover, be emphasized that the BMMSL depend primarily on water exchange with the North Sea. This exchange is controlled by separate conditions, the decisive factor being the sea level difference between the southern Baltic and the North Sea, dependent mainly on the tangential wind stress. Assuming that the tangential wind stress can be approximated by the horizontal component vector of the atmospheric pressure gradient, long-term time series of this phenomenon become available and are computed on the basis of monthly mean atmospheric pressures recorded at selected measuring stations. It is also possible to calculate statistically the isolated influence of the tangential wind stress and the static effect of the atmospheric pressure on the BMMSL.

At the beginning of the computations, monthly mean atmospheric pressure values from the stations in Berufjord, Aberdeen and Utrecht were assumed representative of the Atlantic and North Sea waters adjacent to the Baltic. The Baltic area was represented by Oslo, Copenhagen, Berlin, Kaliningrad, Helsinki, Haparanda and Uppsala. The data covered the years 1901–1937 (Smithsonian Institution, 1944, 1947). The atmospheric pressures were converted to their sea level equivalents.

The first computations were carried out by expanding the atmospheric pressures recorded at the above stations into the EOF amplitudes. The computations were repeated with the introduction of data referring to the Baltic area only. The correlation of EOFs with the sea levels obtained in both cases did not introduce any essential changes into the relationships that could have been obtained between the BMMSL and the individual series of the basic data matrix. This matrix comprised the measurements from the ten named observation stations. The best data correlation was obtained for the station at Uppsala (correlation coefficient −0.650).

The next computations started with the determination of the matrix of gradients between all the stations considered. Depending on their correlation with BMMSL, six data series were distinguished from the matrix and included in further computations. These gradients, together with the stations taken into account in the computations of the mean atmospheric pressure over the Baltic, are summarized in Table 1.

Table 1. Location of measurements introduced into the computations of gradients and mean atmospheric pressures over the Baltic. Measurement period: 1901–1937. Sampling step: 1 month

Gradient computations according to data from	Berlin Copenhagen	Berlin Uppsala	Kaliningrad Uppsala	Kaliningrad Haparanda	Kaliningrad Helsinki	Kaliningrad Oslo
Mean pressure computations according to data from	Copenhagen	Berlin	Kaliningrad	Helsinki	Haparanda	Uppsala

When analysing the data given therein it should be pointed out that the selection of the measuring stations was determined by the possibility of acquiring data for the period covered by the computations. As a result, a fairly uniform distribution along the sea coast was obtained.

The expansion of gradients summarized in Table 1 was carried out using EOF in accordance with formula (3):

$$g_i(t) = \sum\nolimits_{n=1}^{6} \alpha_n(t) d_{ni}, \tag{3}$$

where

$g_i(t)$ – time series of the gradients, $i = 1,\ldots,6$,
$\alpha_n(t)$ – amplitude functions of the gradient matrix,
d_{ni} – local transformation function.

The results of the gradient matrix expansion in amplitude functions $\alpha_n(t)$ and the correlation coefficients between $\alpha_n(t)$ and the BMMSL are given in Table 2. As can be seen, only two first amplitude functions from the set of $\alpha_n(t)$ are distinctly correlated with the BMMSL. This correlation is significant at the level $\alpha = 0.05$ when Student's test is applied to particular series of amplitude functions.

Following the determination of $\alpha_n(t)$ and the mean atmospheric pressure over the Baltic according to Table 1, the effect of the atmospheric pressure field on the BMMSL was evaluated by employing the linear dynamic system of stable parameters and by the spectral density method of determining the system's parameters (Bendat, Piersol, 1986). The computation system proposed was previously employed to several analysis of the forced sea level

Table 2. Results of gradient matrix expansion in EOF amplitudes and the correlation of $\alpha_n(t)$ with the BMMSL

Characteristic	n					
	1	2	3	4	5	6
% of the gradient matrix variance determined by $\alpha_n(t)$	81.0	8.7	8.2	1.2	0.6	0.3
Total % of the gradient matrix variance for increasing number of $\alpha_n(t)$	81.0	89.7	97.9	99.1	99.7	100.0
Correlation coefficient between $\alpha_n(t)$ and BMMSL	0.656	0.319	0.029	-0.079	-0.041	0.086

oscillations, which were cited in the monograph by German and Levikov (1988).

The computations carried out can be characterized in the most concise form by the formula

$$\begin{aligned} \xi_s'(t) &= \int_0^\infty \left[\int_0^\infty H_1(f)\exp(2\pi i f\tau)df\right] \alpha_1(t-\tau)d\tau + \\ &+ \int_0^\infty \left[\int_0^\infty H_2(f)\exp(2\pi i f\tau)df\right] \alpha_2(t-\tau)d\tau + \\ &+ \int_0^\infty \left[\int_0^\infty H_3(f)\exp(2\pi i f\tau)df\right] p(t-\tau)d\tau, \end{aligned} \tag{4}$$

where

$p(t)$ – mean atmospheric pressure over the Baltic.

$$H_j(f) = \frac{G_{yj\cdot lk}(f)}{G_{yy\cdot lk}(f)}, \tag{4a}$$

where

$H_j(f)$ – frequency response function of the output of system y with the j-th input; the Fourier transform $H_j(f)$ is the weighting function $h(\tau)$ computed in real time steps, τ,

$G_{yj\cdot lk}(f)$ – conditioned cross-spectral density function of output y with input j, and eliminated operation of inputs l and k,

$G_{yy\cdot lk}(f)$ – conditioned autospectral density function of the output process y with eliminated operation of inputs l and k.

Characteristic of the method assumed to estimate the parameters of the system is the expectation of a meaningful noise level. It was therefore essential that the series $h(\tau)$ with a size obtained according to the assumed maximum autocorrelation interval be cut. The stochastic characteristics of

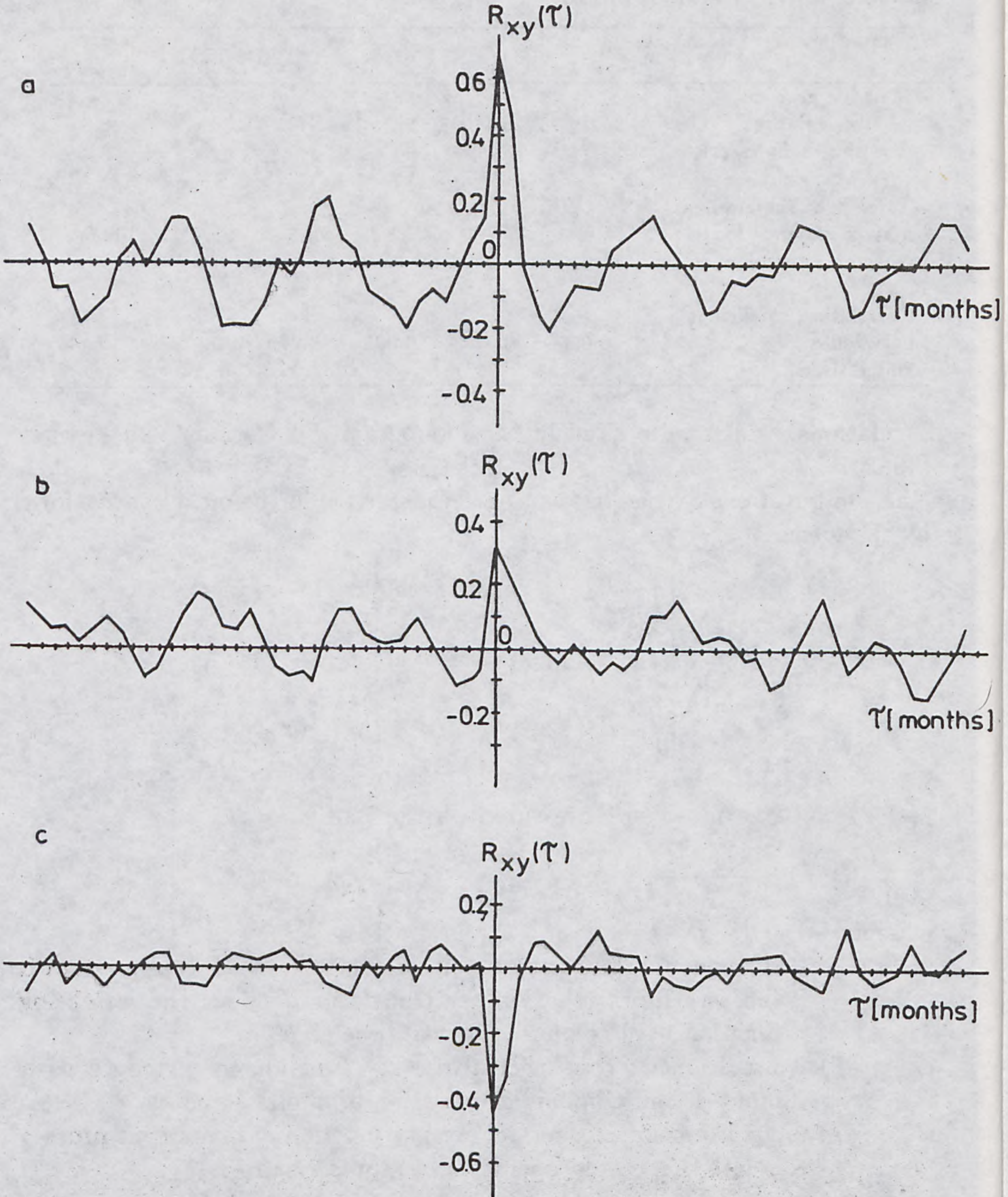

Fig. 3. Cross-correlation functions of the atmospheric pressure field elements and BMMSL. $\alpha_1(t)$ – BMMSL (a), $\alpha_2(t)$ – BMMSL (b), mean atmospheric pressure over the Baltic – BMMSL (c). Collinearity among the input series is not excluded

the computations, *i.e.* the auto- and cross-correlation functions, are shown in Figures 2 and 3. The use of the values of $h(\tau)$ were followed by truncation which equated all functions $h(\tau)$ to zero for $\tau > 1$. The assumed truncations are justified by the dynamic effect of the atmospheric pressure field elements on the BMMSL.

After having determined $h_j(\tau)$, the computations of $\hat{\xi}'_s(t)$ proved effective. The multiple correlation coefficient for three inputs was 0.897, and 80.4% of the BMMSL variance could be determined. The r.m.s. error of the hindcast computations was 6.5 cm. BMMSL according to the measurements and computations during the first half of the period under analysis are shown in Figure 4.

Table 3. Isolated effect of the atmospheric pressure field elements on the BMMSL

Atmospheric pressure field element	σ_{SM} [cm]	$\hat{\xi}_{\min}$ $\hat{\xi}_{\max}$ [cm]	Ra [cm]	$\bar{e}$ [cm]
Amplitude function $\alpha_1(t)$	11.5	-43.3 29.5	72.8	9.9
Amplitude function $\alpha_1(t)$	5.8	-24.0 5.6	29.6	13.7
Mean pressure over the Baltic, $p(t)$	1.4	-13.6 -4.1	9.5	14.1

where

σ_{SM} – standard deviation of the BMMSL oscillations due to the isolated dynamic effect of an atmospheric pressure field element,

$\hat{\xi}_{\min}$, $\hat{\xi}_{\max}$ – maximum and minimum sea levels of the oscillations considered,

Ra – range of the oscillation interval,

$\bar{e}$ – r.m.s. error in the BMMSL determination by an isolated atmospheric pressure field element.

The results obtained were computed in a correlated input system, which means that the weighting functions represent the isolated operation of the individual series. The operation of the elements of the superimposition formula (4) reflects the dynamic effect of the individual atmospheric pressure field components on the BMMSL. The relevant data are presented in Table 3.

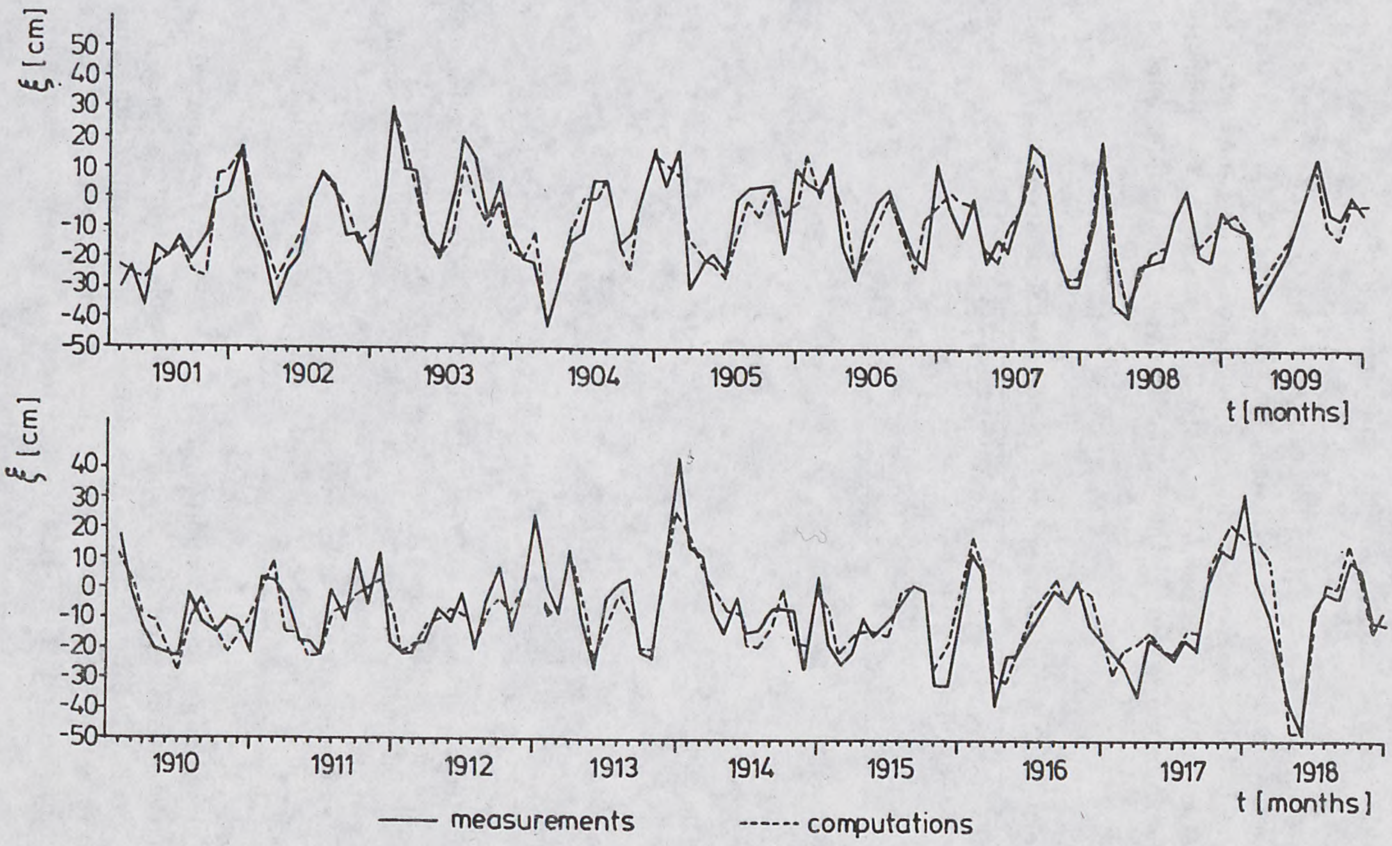

Fig. 4. Baltic monthly mean sea levels according to the measurements and computations based on formula (4)

The computations were verified by employing another method used for estimating the parameters of a linear dynamic system (Kalman, 1958). The basic matrix of the input processes was established by employing input processes and a dynamic system in accordance with the previous assumptions in formula (4). By applying EOF, the matrix **X** of the amplitude functions $x_j(t)$ was obtained. Condition $\mathbf{Var}(\mathbf{X}) = \mathbf{I}$ was fulfilled. After cross-validation, the final matrix of the system inputs and the vector of structural parameters were obtained (Golub *et al.*, 1979; Lachenbruch and Goldstein, 1979). The difference in the explained variance values between the computations obtained by estimation and those obtained using formula (4) was less than 1%.

4. Analysis of the computation results

When considering the reasons for the occurrence of the annual BMMSL oscillation period it should be emphasized that this phenomenon is associated with the annual solar tide, Sa (Maksimov, 1970). In the Baltic Sea literature to date, the wind stress influence on the BMMSL oscillations is commonly determined by single linear characteristics, for example by the regression connected with one horizontal component of atmospheric pressure (Lazarenko, 1961; Lisitzin, 1962). The computations reported here relate the BMMSL directly to the whole atmospheric pressure and wind fields over the Baltic. The effect of these fields is principally manifested by the operation of the tangential wind stress vectors monitoring the water exchange with the North Sea. The operation of the EOF amplitude functions of the field is presented in Table 3 and Figure 3, where the cross-correlation functions display the annual period for the BMMSL generated by $\alpha_1(t)$ and not distinctly by $\alpha_2(t)$. It is in the activity of these factors that the main reason for the annual BMMSL oscillation should be sought. Needless to say, it is not possible to separate exactly the slight effect of Sa from that of $\alpha_1(t)$ and $\alpha_1(t)$. For comparison, it is worth mentioning that according to the equilibrium theory, the range of the combined annual and semiannual tides is 2.0 cm at latitude 60° (Pattulo *et al.*, 1955).

The static atmospheric pressure influence on the annual character of the BMMSL changes is of negligible significance in view of the generally slight effect of this factor together with the lack of a distinct annual period of the averaged pressure field oscillations. The slight effect of such elements of the water budget as aerial precipitation, evaporation and river inflows on the oscillations seems worth pointing out. The presence of a weak steric component is also known.

As a result of all these factors, most importantly the functions $\alpha_1(t)$ and $\alpha_2(t)$, the lowest BMMSL occur in April and May and the highest in

August and September. The amplitude of the annual BMMSL oscillation period is 9.4 cm according to harmonic analysis computations. All the autocorrelation and cross-correlation functions indicated a meaningful noise in the series tested.

The occurrence of the semiannual period in the BMMSL oscillations has already been evidenced by the computations of the auto-correlation function presented in Figure 2. Such level changes tend to fade in the noises of the phenomenon. The occurrence of Ssa has commonly been accounted for in the literature by the effect of the semiannual solar tide (Karklin, 1967).

The harmonic analysis computations carried out in this study yield an amplitude of these oscillations of 5.5 cm. The computed oscillation maxima occur in January and July, and the minima in April and October, which is in accordance with the static theory of solar tides. The occurrence of a faint semiannual tide in the atmosphere has been evidenced by the computations of functions $\alpha_1(t)$ and $\alpha_2(t)$ by the method of harmonic analysis.

When the BMMSL variations are considered in terms of the time series run and not of the periodical structure, conclusions can be drawn as to the decisive wind field effect on the analysed oscillations, as shown in Table 3 and Figure 4. The direct effect of this field was defined in Table 3 as standard deviations of the level variations due to the isolated exciting forces. The Baltic level changes resulting from the static influence of the atmospheric pressure field estimated relative to the global atmospheric pressure distribution were analysed by Pattulo *et al.* (1955). In the present paper, the standard deviation of the sea level variations, amounting to 1.4 cm, determines the isolated local static effect of the atmospheric pressure variations over the Baltic. Together with the influence of wind stress, the atmospheric pressure field determines the sea level changes which have a standard deviation of 13.2 cm.

As already mentioned, according to the commonly cited inverted barometer rule, an atmospheric pressure change of 1 hPa corresponds to a 1 cm sea-level change of the opposite sign. The relevant data for the Baltic were obtained on the basis of the relationship between the isolated effect of the averaged pressure field over the basin and the BMMSL changes. The value of -0.16 cm was obtained in this way.

Water temperature in the basins and other factors of minor importance affecting the BMMSL were not taken into account in the time series of the inputs to the linear system of stable parameters. The collinearity of these elements with the input series could have only marginally influenced the computation results presented here. The very small influence of tides of a period longer than one year has not been considered.

The nature of BMMSL variations and the computed forcing characteristics indicate that the assumed data sampling step of 1 month could be replaced by a 14-day step which would probably yield a better representation of the input processes. Data of this kind have not been processed yet.

5. Conclusions

This study deals with the influence of the atmospheric pressure field over the basin on the seasonal changes of the Baltic mean sea level. The use of EOF together with the linear dynamic system of stable parameters has resulted in the separation of the static influence of the mean atmospheric pressure field from the action of the wind field. The static influence of the local atmospheric pressure is presented as the forcing of the sea level changes which amounts to 0.9% of the BMMSL variance. The action of the atmospheric pressure field, including wind stress, determines 80.4% of the seasonal sea level variance. A change in the mean atmospheric pressure over the Baltic of 1 hPa corresponds to a sea level change of 0.16 cm of the opposite sign.

The estimation of the linear dynamic system parameters was performed by two methods yielding practically the same results. The mean monthly characteristics assumed in the computations are based on data from a period of 37 years and are reliable.

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